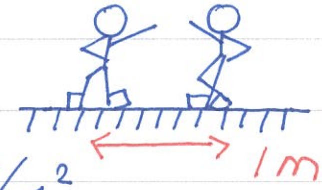
豪豬筆記

HH0107 Why are most materials charge neutral?

The short answer to this question is ... Hum, not really. The longer answer involves how to estimate the strength of Coulomb interaction.

Let us estimate the gravitational force between Jack and Jill who are 1 m apart.

Suppose their body mass is the same, $m = 60\,kg$. The G force between them is $\rightarrow$

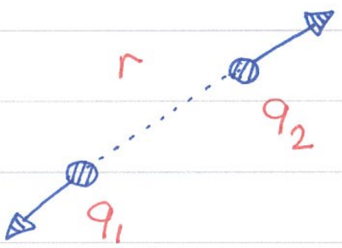


$$F = \frac{Gm^2}{r^2} = 6.67\times10^{-11}\times(60)^2/1^2$$

The force is roughly $F \sim 10^{-7}\,N$ ← very weak

The Coulomb force between two charged objects takes the similar form,

$$\boxed{F = k\frac{q_1 q_2}{r^2} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}}$$



where $k = \frac{1}{4\pi\epsilon_0} = 8.99\times10^{9}\ \frac{N\cdot m^2}{C^2}$

It is not too bad to assume that we are made of water molecules.

$$\frac{60\,kg}{18\,g} \cong 3.33\times10^{3} \rightarrow 10^{3}\ \text{mole}$$

Suppose each water molecule just looses $1e^-$. Then, Jack/Jill will carry some charges....

$$Q = 1.6\times10^{-19}\times10^{3}\times6\times10^{23} \sim 10^{8}\ (C)$$

The Coulomb force can be estimated easily,

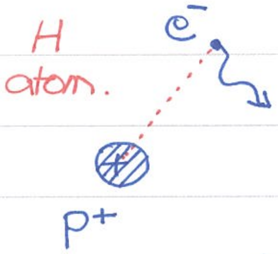
$$F = \frac{1}{4\pi\epsilon_0}\frac{Q^2}{r^2} = 8.99\times10^{9}\cdot\frac{(10^8)^2}{1^2} \approx 10^{26}\,N\ !!$$

Even if we just loose/gain $1e^-$ for each composing molecule, we will be doomed....

$$F_G \sim 10^{-7}\,N, \quad F_c \sim 10^{26}\,N \longleftarrow \text{HUGE.}$$

To avoid the strong Coulomb interaction, most stable materials are charge neutral.

It is also fun to compare F_G and F_c at atomic scale.



$$\frac{F_c}{F_G} = \frac{\frac{1}{4\pi\epsilon_0}\frac{e^2}{\cancel{r^2}}}{G\frac{m_p m_e}{\cancel{r^2}}} = \frac{1}{4\pi\epsilon_0 G}\cdot\frac{e}{m_p}\cdot\frac{e}{m_e}$$

$$\longrightarrow \frac{F_c}{F_G} \approx 10^{20}\cdot 10^{8}\cdot 10^{11} = \underline{\underline{10^{39}}}$$

Because the above ratio is huge, we don't need to consider gravitational interaction when exploring the structure of atoms! It is important to emphasize that the above estimates are not merely from the ratio of Coulomb and Newton constants k/G because it's not dimensionless.

⓪ Understanding electromagnetic interactions. It turns out the field theory is the mother language to describe EM interactions.

$$\boxed{\begin{aligned} &\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0} && \vec{\nabla}\cdot\vec{B} = 0 \\ &\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} && \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \end{aligned}}$$

These are the famous Maxwell equations

To describe the EM interactions, 2 vector fields are needed:

$$\boxed{\vec{E} = (E_x, E_y, E_z) \text{ and } \vec{B} = (B_x, B_y, B_z)}$$

Each component may depend on x, y, z, t.

$$E_x = E_x(x, y, z, t) \rightarrow \frac{\partial E_x}{\partial x}, \frac{\partial E_x}{\partial y}, \frac{\partial E_x}{\partial z}, \frac{\partial E_x}{\partial t} \leftarrow \text{4 derivatives.}$$

Wow, $(3+3) \times 4 = \underline{24}$ first derivatives in total. You may guess 24 equations are necessary for a complete description. Nope ... Count again and you should find out that there are only $1+3+1+3 = \underline{\underline{8}}$ Maxwell eqs. Think about this interesting mismatch ☺

Don't be scared by the scary notations.

Divergence: $\boxed{\vec{\nabla} \cdot \vec{E} \equiv \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}}$ ← scalar.

The above definition can be understood as

$$\vec{\nabla} \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \rightarrow \vec{\nabla} \cdot \vec{E} = \frac{\partial}{\partial x} E_x + \frac{\partial}{\partial y} E_y + \frac{\partial}{\partial z} E_z.$$

Curl: $\boxed{\vec{\nabla} \times \vec{E} \equiv \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)}$

↑ vector

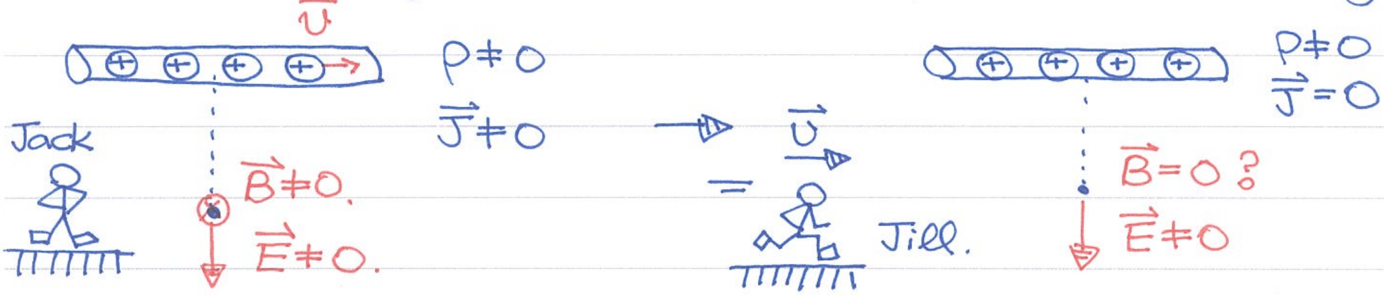
Similarly, the above definition can be understood as the vector product of $\vec{\nabla}$ & $\vec{E}$.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}, \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

The Maxwell equations contain two important structures : (1) source terms (2) fields coupled through time derivatives.

$$\boxed{\begin{array}{ll} \text{charge density } \rho(\vec{r},t) & \longrightarrow \vec{E}(\vec{r},t) \\ \text{current density } \vec{J}(\vec{r},t) & \longrightarrow \vec{B}(\vec{r},t) \end{array}}$$

The connections between the sources and the corresponding fields seem very reasonable. But, it can be confusing.



Back to the Jack-and-Jill problem. Because $\rho \neq 0, \vec{J} \neq 0$, Jack should observe non-vanishing $\vec{E}$ and $\vec{B}$ fields. But, in Jill's frame, the current density is zero. Does this mean that the magnetic field vanishes here ?

Now we turn to the coupling between $\vec{E}$ & $\vec{B}$ fields. If the fields are static, i.e. $\frac{\partial \vec{E}}{\partial t} = 0, \frac{\partial \vec{B}}{\partial t} = 0$

The Maxwell equations decouple into two groups. The $\vec{E}$ field and the $\vec{B}$ field do not talk to each other anymore.

$$\boxed{\begin{aligned} \vec{\nabla}\cdot\vec{E} &= \frac{\rho}{\epsilon_0} \\ \vec{\nabla}\times\vec{E} &= 0 \end{aligned}} \;\neq\; \boxed{\begin{aligned} \vec{\nabla}\cdot\vec{B} &= 0 \\ \vec{\nabla}\times\vec{B} &= \mu_0\vec{J} \end{aligned}}$$

Of course, for consistency, the sources $\rho = \rho(\vec{r})$ $\vec{J} = \vec{J}(r)$ should be static as well.

Most of the time, $\vec{E}$ & $\vec{B}$ fields vary with time.
From the Maxwell equations,

$\frac{\partial \vec{B}}{\partial t}$ generates $\vec{E}$, $\frac{\partial \vec{E}}{\partial t}$ generates $\vec{B}$

It brings out the interesting consequence $\rightarrow$
$\vec{E}$ and $\vec{B}$ can live without the source terms.

$$\boxed{\begin{aligned} &\vec{\nabla}\cdot\vec{E} = 0 &\qquad &\vec{\nabla}\cdot\vec{B} = 0 \\ &\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} &\qquad &\vec{\nabla}\times\vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} \end{aligned}}$$

$(\vec{E}, c\vec{B}) \rightarrow (c\vec{B}, -\vec{E})$
invariant.

The symmetry between the electric and the magnetic fields becomes manifest in the free space ($\rho=0, \vec{J}=0$).

It is also interesting to observe how the principle of superposition emerges. Suppose two sets of solutions $(\vec{E}_1, \vec{B}_1)$ and $(\vec{E}_2, \vec{B}_2)$ satisfy the Maxwell equations in the free space. It is easy to show that

$$\vec{\nabla}\times(\vec{E}_1+\vec{E}_2) = \vec{\nabla}\times\vec{E}_1 + \vec{\nabla}\times\vec{E}_2 = -\frac{\partial \vec{B}_1}{\partial t} - \frac{\partial \vec{B}_2}{\partial t} = -\frac{\partial}{\partial t}(\vec{B}_1+\vec{B}_2).$$

The same holds for the other equations. Thus, linear superposition $(\vec{E}_1+\vec{E}_2, \vec{B}_1+\vec{B}_2)$ also satisfies the Maxwell equations. Note that this principle of superposition no longer works when the sources are present.

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