HH0118 Magnetic Dipole

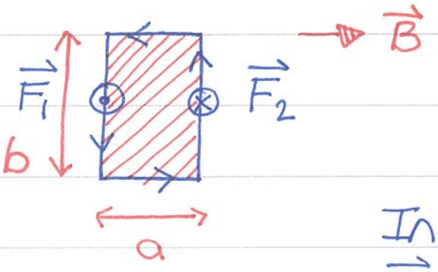
In previous lecture, we learned the lesson:

→ ONE $\vec{B}$ field, but TWO magnetic forces

To understand the Type II magnetic force, it is important to study the magnetic dipole $\vec{\mu}$.

⓪ A current loop.

Consider a rectangular loop carrying current I in a uniform $\vec{B}$ field.



$$|\vec{F_1}| = |\vec{F_2}| = IbB$$

And, $\vec{F_1} + \vec{F_2} = 0$ → no net force.

In fact, for an arbitrary loop in uniform $\vec{B}$ field, the net force is always zero ☺

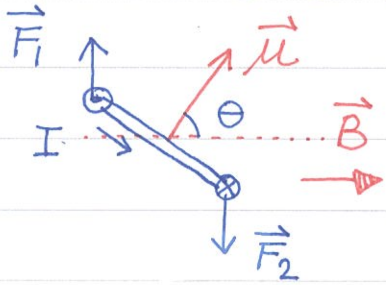
$$\vec{F} = I \oint d\vec{\ell} \times \vec{B} = I \left(\oint d\vec{\ell} \right) \times \vec{B} = 0 \;!!$$

But, there's a torque acting on the current-carrying loop.

$$\tau = F_1 \cdot \frac{a}{2} + F_2 \frac{a}{2} = IabB = IAB$$

$A \equiv a \cdot b$ is the area of the loop.

Now consider a tilted loop as shown below. It is inspiring to define the magnetic dipole moment $\vec{\mu}$ as



$$\vec{\mu} \equiv I\vec{A}$$

It is easy to compute the total torque on the loop.

$$\tau = F_1 \sin\theta \frac{a}{2} + F_2 \sin\theta \frac{a}{2} = IabB\sin\theta = \mu B \sin\theta$$

The above relation can be written down in vector form,

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

It's quite cute that $\vec{B}$ field tends to align the dipole moment $\vec{\mu}$ ☺

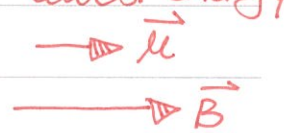
The potential energy associated with $\vec{\mu}$ is

$$\Delta U = -\int \vec{\tau}\cdot d\vec{\theta} = -\int_{\theta_1}^{\theta_2} -\mu B \sin\theta \, d\theta = -\mu B\cos\theta \Big|_{\theta_1}^{\theta_2}$$

Choose $U(\theta=\frac{\pi}{2})=0$ for convenience,

$$\boxed{U(\theta) = -\mu B\cos\theta = -\vec{\mu}\cdot\vec{B}}$$

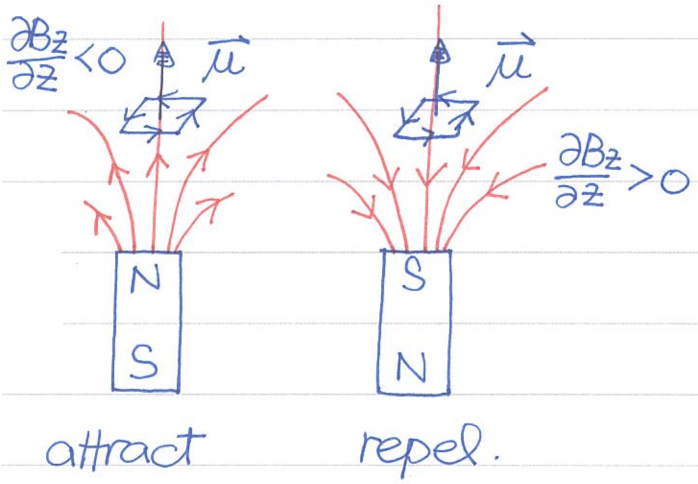
lowest energy.



When $\vec{\mu}$ aligns with $\vec{B}$, it reaches the lowest energy ☺

② Magnetic force in non-uniform field. In a nonuniform field, the net force on a magnetic dipole is no longer zero ...

$$\boxed{\vec{F} = -\vec{\nabla}U = \vec{\nabla}(\vec{\mu}\cdot\vec{B})}$$

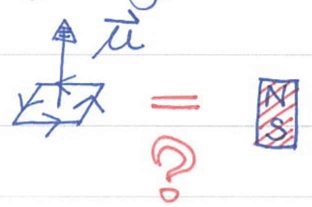


Consider a dipole moment near a magnet as shown.

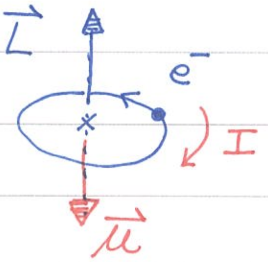
$$F_z = \frac{\partial}{\partial z}(\mu B_z) = \mu \frac{\partial B_z}{\partial z}$$

The force depends on the sign of $\partial B_z/\partial z$!

Compare the above results with the magnet. It is inspiring to draw the analogy between current loop and the usual magnet. Are they completely equivalent? They feel the same magnetic force (Like poles repel while opposite poles attract.) and they generate the same $\vec{B}$ field. It is tempting to vote for their equivalence. But, although both of them can be described by $\vec{\mu}$, the microscopic origins are different — quantum mechanics is needed here.



① The atomic magnet. Let us consider an electron in circular motion. The magnetic dipole moment is $\mu = IA = \dfrac{e\pi r^2}{T}$

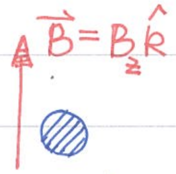


Express the period in terms of speed $T = 2\pi r/v$,

$$\mu = \frac{e\pi r^2}{2\pi r/v} = \frac{1}{2} erv = \frac{e}{2m}(mvr) = \frac{e}{2m} L \quad \leftarrow \text{angular momentum.}$$

Note that the charge of an electron is $-e$. The above relation in vector form should be

$$\boxed{\vec{\mu} = -\frac{e}{2m}\vec{L}} \qquad \vec{\mu} \propto \vec{L}$$



an atom in $\vec{B}$ field.

Now consider an atom in the $\vec{B}$ field along the z axis. The potential energy due to the $\vec{B}$ field is

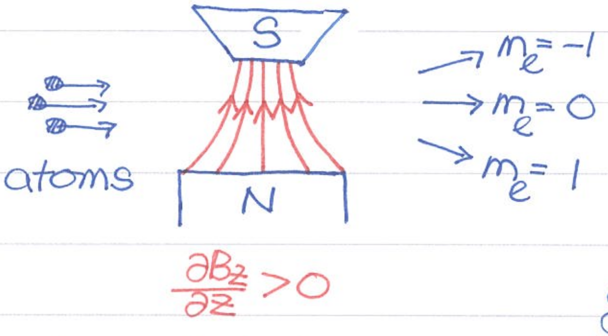
$$\boxed{U = -\vec{\mu}\cdot\vec{B} = -\mu_z B_z = \frac{e}{2m} L_z B_z}$$

In quantum mechanics, L_z is not continuous,

$$L_z = m_\ell \hbar, \quad m_\ell = 0, \pm 1, \ldots \quad \rightarrow \quad \boxed{U = \left(\frac{e\hbar}{2m}\right) m_\ell B_z = \mu_B m_\ell B_z}$$

It is rather remarkable that the tiny magnet indeed exists – it is the Bohr magneton $\mu_B = e\hbar/2m$!

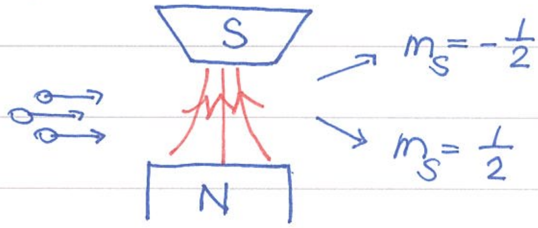
Let us consider the famous Stern-Gerlach experiment. The magnetic force along the z axis



$$F_z = -\frac{\partial U}{\partial z} = -\mu_B \frac{\partial B_z}{\partial z} \cdot m_\ell$$

In the experimental setup, the gradient $\partial B_z/\partial z > 0$. Thus, $m_\ell = 1$ atoms feel downward force while $m_\ell = -1$ atoms feel the upward force. The force on $m_\ell = 0$ atoms vanishes.

⊘ Electron spin. In 1922, Stern and Gerlach let silver atoms passing through the magnet with non-uniform $\vec{B}$ field. They found the atom beam splits into TWO.



But, this outcome is puzzling....
The ground state of Ag atom carries no angular momentum, i.e. $m_\ell = 0$ → no deflection expected.

In 1925, Goudsmit and Uhlenbeck proposed that the electron carries intrinsic angular momentum called spin $\vec{S}$:

$$S_z = m_s \hbar, \quad m_s = \frac{1}{2}, -\frac{1}{2}$$

orbital $m_\ell = 0, \pm 1, \cdots$ (integer.)
spin $m_s = \pm\frac{1}{2}$

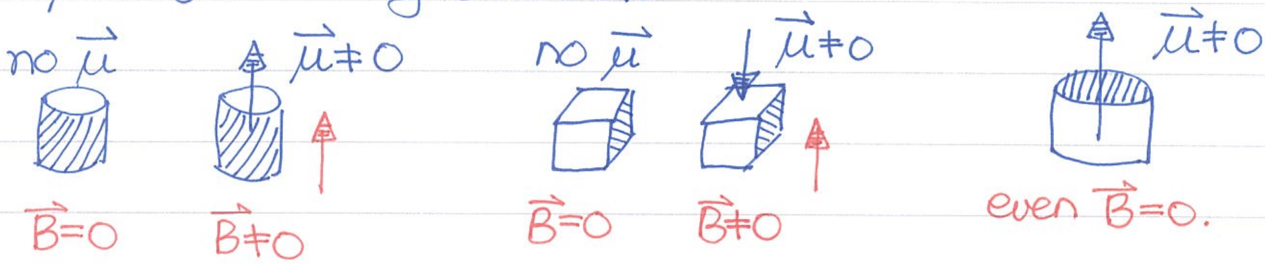
Not surprisingly, spin is related to the magnetic dipole moment,

$$\vec{\mu} = -g\mu_B(\vec{S}/\hbar), \quad \text{where } g \approx 2$$

← $\vec{\mu} = -\mu_B(\vec{L}/\hbar)$ for orbital part

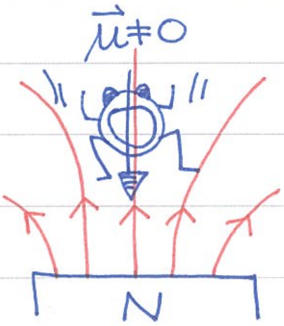
Because the electron is a point, the origin of spin is NOT due to its spinning motion. The origin of $\vec{S}$ was later explained by Dirac, using a relativistic quantum theory to keep the Lorentz invariance.

⊘ Magnetic materials. We can classify the materials by their responses in a magnetic field.



"paramagnetic" "diamagnetic" FERROmagnetic

The water is weakly diamagnetic. Most animals are mainly made of water molecules → most animals shall be diamagnetic ☺



$$F_z = \mu_z \cdot \frac{\partial B_z}{\partial z}$$

$\mu_z < 0$, diamagnetic

Thus, $F_z > 0$ providing a repulsive force. If this force is large enough, the poor animal will float.

Geim demonstrated the magnetic levitation of frog and received the 2000 Ig Nobel Prize. Later, Geim was awarded the 2010 Nobel Prize in Physics jointly with Novoselov "for groundbreaking experiments regarding the two dimensional material graphene" — also a strong diamagnetic material ☺

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