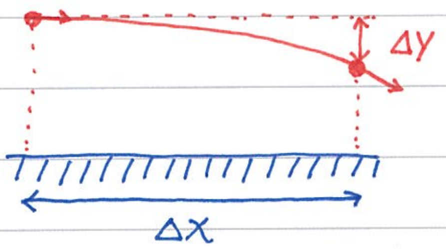
HH0128 What Is the Nature of Light?

Scientists have been confused by the nature of light for centuries. Is light made up of tiny particles? OR, is light some sort of wave? The answer turns out to be highly non-trivial ☺

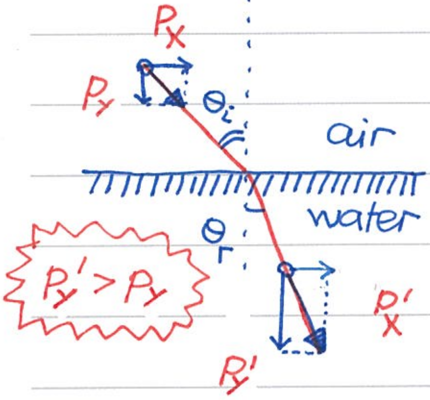
(1) Newton's theory of light: In 1704, Newton published the book "Opticks", 17 years after his great work "Principia". He proposed that light is made up of tiny particles. One can estimate the deflection of light in the gravitational field.



$$\Delta x = c\,\Delta t, \quad \Delta y = \frac{1}{2} g (\Delta t)^2$$

Suppose the time interval $\Delta t = 1\,ms = 10^{-6}\,s$. When light travels $\Delta x = 300m$, the vertical displacement is very small, $\Delta y \approx 5\times10^{-12}\,m$, in comparison.

A key property of the particle theory is refraction. Because the tiny particle of light has mass, it experiences an attractive force from air to denser media (like water).



The momentum in the horizontal direction remains the same (no force ☺)

$$p_x = p_x' \longrightarrow p\sin\theta_i = p'\sin\theta_r$$

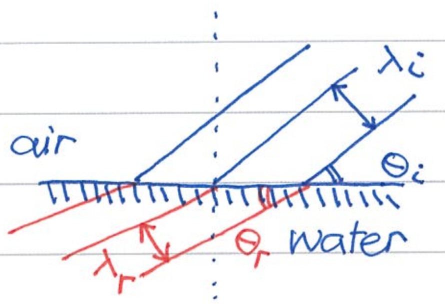
Thus, the speed of light in water should be faster!

$$\frac{p'}{p} = \frac{mc'}{mc} \longrightarrow \frac{c'}{c} = \frac{\sin\theta_i}{\sin\theta_r} > 1$$

Snell's law explained ☺

Newton regarded the above explanation for the Snell's law as one of his triumphs. He stated in the book "Opticks" with the words : I take this to be a very convincing argument of the full truth of this proposition.

On the other hand, Huygens propsed light is a longitudinal wave propagating in ether. His theory is mainly kinematic and its scope is largely restricted to the geometric optics. But, it provides a different explanation of Snell's law.



From the geometry, it is easy to see

$$\frac{\lambda_i}{\lambda_r} = \frac{\sin\theta_i}{\sin\theta_r}$$

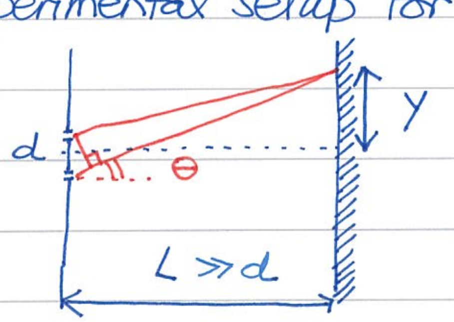
Because the frequency does not change in refraction, $f_i = f_r$. Thus, the speed of light in water is expected to be slower than that in the air,

$$\frac{c'}{c} = \frac{f_r \lambda_r}{f_i \lambda_i} = \frac{\sin\theta_r}{\sin\theta_i} < 1$$

opposite to Newton's prediction !!

Who is right ? Around 1850, Foucault measured the speed of light in water and found $c' < c$. This experiment is decisive and kills Newton's particle theory of light.

Ø Young's double-slit experiment : Consider the following experimental setup for light interference. The constructive interference occurs when



$$d \sin\theta = m\lambda \qquad m = 0, \pm 1, \cdots$$

← criterion for intensity maxima.

Making use of the geometric relation,

$$\frac{y}{L} = \tan\theta \approx \sin\theta = \frac{m\lambda}{d} \longrightarrow \boxed{y_m = m\frac{\lambda L}{d}}$$

One would observe equally spaced maxima with adjacent distance $\Delta y = \lambda L/d$.

Let us try to understand Young's experiment by linear superposition. Let us assume that the electric fields at the observation points are sinasoidal in time

$$E_1 = E_0 \sin(\omega t + \phi_1), \quad E_2 = E_0 \sin(\omega t + \phi_2)$$

The phases ϕ_1, ϕ_2 are different due to different paths!

$$\begin{matrix} \phi_1 = k r_1 \\ \phi_2 = k r_2 \end{matrix} \longrightarrow \Delta\phi = \phi_2 - \phi_1 = k(r_2 - r_1) = k\Delta r$$

The path difference $\Delta r = d\sin\theta$ and the wave number $k = 2\pi/\lambda$. Thus, the phase difference $\Delta\phi$ depends on the angle θ,

$$\boxed{\Delta\phi = k\Delta r = \frac{2\pi}{\lambda} d\sin\theta = 2\pi\frac{d\sin\theta}{\lambda}}$$

If the slit separation d is much smaller than the distance L, the $\vec{E}$-fields are nearly parallel. The summation of the fields becomes rather simple

$$E = E_1 + E_2 = E_0\sin(\omega t + \phi_1) + E_0\sin(\omega t + \phi_2)$$

$$\longrightarrow \boxed{E = 2E_0\cos\left(\tfrac{1}{2}\Delta\phi\right)\sin(\omega t + \bar{\phi})} \quad \leftarrow \bar{\phi} = \tfrac{1}{2}(\phi_1 + \phi_2)$$

The intensity of light is just the averaged Poynting vector $\longrightarrow I = \langle S \rangle = \frac{1}{\mu_0}\langle E(t)B(t)\rangle$.

For electromagnetic waves, $B(t) = E(t)/c$,

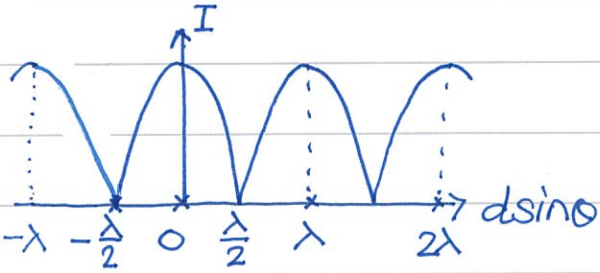
$$I = \frac{1}{\mu_0 c}\langle E^2 \rangle = c\epsilon_0 \cdot 4E_0^2 \cos^2\left(\tfrac{1}{2}\Delta\phi\right)\langle \sin^2(\omega t + \bar{\phi})\rangle$$

The time average can be computed by integration,

$$\langle \sin^2(\omega t+\bar{\phi})\rangle = \frac{1}{T}\int_0^T \sin^2(\omega t+\bar{\phi})\,dt = \frac{\pi}{\omega T} = \frac{1}{2}$$

Note that the phase difference $\Delta\phi = 2\pi\,(d\sin\theta/\lambda)$,

$$I = 2c\epsilon_0 E_0^2 \cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right) = I_{max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)$$



max : $d\sin\theta = 0, \pm\lambda, \pm 2\lambda, \cdots$

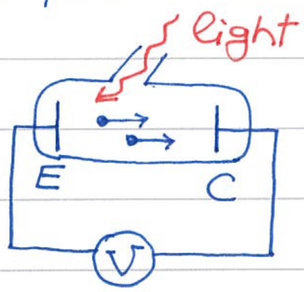
min : $d\sin\theta = \pm\frac{1}{2}\lambda, \pm\frac{3}{2}\lambda, \cdots$

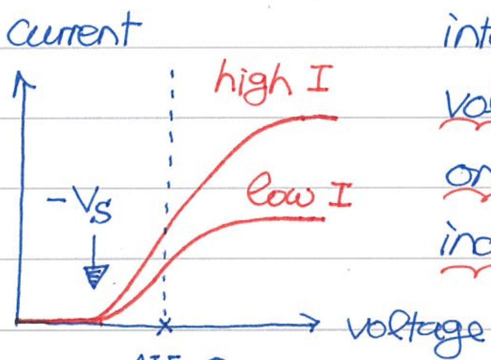
Agree with previous results. With a bit further analysis, one can show that the intensity for E_1 and E_2 is the same,

$$I_1 = I_2 = \frac{1}{2}c\epsilon_0 E^2 = \frac{1}{4}I_{max}$$

← Can you explain this by simple argument ?

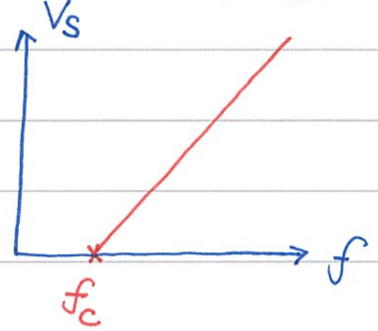
Ⓓ Emergence of photons : The Maxwell's equations provide solid understanding of light as the EM wave. But, the puzzle about the nature of light goes on... In the photo-electric experiment, one finds that the data can be explained by photons — the particle theory is back ! It is quite





interesting that the stopping voltage V_s only depends on the frequency of the incident light, not on the intensities !

Plot the stopping voltage V_s for different frequencies:



There exists a cutoff freq. f_c!
The frequency of the incident light must be greater than f_c to observe the photoelectric current ☺

Einstein proposed a simple picture of photons to explain the photoelectric effect:

light ≡ →→→ (photons)

The energy of a photon is

$$E = hf$$

The maximum kinetic energy of the electron knocked out by a photon satisfies the Einstein's equation:

$$hf = K_{max} + \phi$$

← ϕ is the work function.

The maximum kinetic energy is related to the stopping voltage.

$$eV_s = K_{max}$$

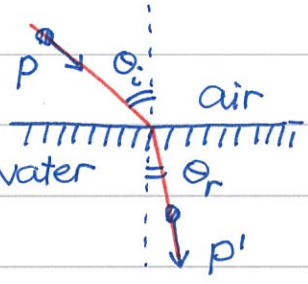
The stopping voltage corresponds to the potential difference so that all electrons loose their kinetic energy. → No current is flowing anymore ☺.

Einstein also proposed the following relation for momentum:

$$p = \frac{h}{\lambda} \quad \rightarrow \quad E = hf = \frac{h}{\lambda} c = pc$$

relativistic particle without mass!

Compared with $E^2 = p^2c^2 + m^2c^4$ for relativistic particles, photons are relativistic particles with zero rest mass. We can also revisit the refraction problem. The horizontal momentum is conserved: $p\sin\theta_i = p'\sin\theta_r$



$$\frac{p'}{p} = \frac{\lambda_i}{\lambda_r} = \frac{\sin\theta_i}{\sin\theta_r}$$

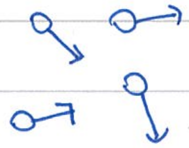
Snell's law for photons ☺

Because the frequency remains the same,

$$\frac{c'}{c} = \frac{\cancel{f_r}\lambda_r}{\cancel{f_i}\lambda_i} = \frac{\sin\theta_r}{\sin\theta_i} < 1$$

different from Newton's particle.

The energy-momentum relations proposed by Einstein make photons different from Newton's particle theory of light. In fact, these relations mark the dual nature of particle and wave in light. Later, we will similar duality for other objects like electrons. It turns out that the dual nature of particle and wave is intrinsic in the quantum world ☺



particle

$$E = hf$$

$$p = \frac{h}{\lambda}$$

wave

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