Final for General Physics I

Date: January 12, 2014

(1) Please do not flip the sheet until instructed.
(2) Write as neat as possible, many thanks!
(3) Make the logical flows in your answers clear.
(4) Good luck for all hard-working students!

Lecturer: Hsiu-Hau Lin

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1. Energy propagation (20%) A right-moving wave is propagating in an elastic rope with tension T and mass density ρ. Its wave function is given by

$$u(x,t) = A \exp\left[-\frac{1}{2a^2}(x-vt)^2\right],$$

where $v = \sqrt{T/\rho}$ is the wave speed and A, a are some constants. Find the power of energy propagation $P(x,t)$ and sketch its profile at $t=0$.

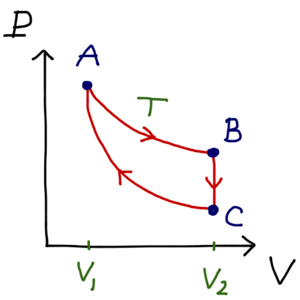
2. Forecast uncertainty (20%) In computer science, due to the binary feature of the hardware, the information entropy is often defined as

$$\sigma = \sum_s -P_s \log_2 P_s,$$

where P_s is the probability for state s. Treat the weather tomorrow as a random variable X with only three possible outcomes: C (cloudy), R (rainy) and S (sunny). The weather forecast claims that the probability for a sunny day tomorrow is 1/2 and those for the cloudy and the rainy are 1/4, 1/4 respectively. Compute the information entropy σ. In principle, what is the maximum information entropy $\sigma_{\max}$ for the CRS weather model?

3. Maxwell speed distribution (20%) A monoatomic ideal gas is confined in two dimensions. The mass of the gas molecule is m and the ideal gas is in thermal equilibrium with a reservoir of temperature T. Find the velocity distribution $P(\boldsymbol{v})$ of the gas molecule? If we do not care about the directions of the velocity, what is the speed distribution $P(v)$ for the ideal gas?

4. Heat engine (20%) Consider a heat engine consists of one mole of monoatomic ideal gas with the three-step cycle as shown in the figure. The heat engine starts from state A, to state B and state C, then returns to the original state A. The first process $A \to B$ is isothermal at temperature T from a smaller volume $V_1 = v$ to the larger $V_2 = 8v$; the second process $B \to C$ occurs at constant volume and the third process $C \to A$ is adiabatic. Find the efficiency ϵ of the heat engine in terms of T, v and the ideal-gas constant R.



(Bonus 10%) Compute the corresponding Carnot efficiency ϵ_C in the same working temperature range. What can you say when comparing ϵ and ϵ_C?

5. Entropy (20%) Write a short essay (less than two pages in your answer booklet) on *"Information and thermal entropies"*. You may want to start from the definitions for the information entropy and the thermal one. Take ideal gas as an example to demonstrate their equivalence. or, you can try to find their relation in general cases. Elaborate on the validity of their equivalence in equilibrium and/or out of equilibrium. It is important to get the logic straight and may be helpful to provide concrete examples when necessary.

6. Wave attenuation (Bonus 20%) Travelling waves attenuate in real life because energy dissipation is inevitable. When the dissipative power is small, the wave equation takes the following form,

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} + 2\beta v^2 \frac{\partial u}{\partial x},$$

where v is the wave speed and β is some positive constant. A harmonic travelling wave with angular frequency ω is conveniently described by taking the real part of the complex exponential function,

$$u(x,t) = \Re\left[e^{ikx - i\omega t}\right],$$

where the angular frequency ω is real but the wave number k needs to be determined and can be complex. Assuming $\omega \gg \beta v$, find the complex wave number k and thus the real wave function $u(x,t)$. Can you see the attenuation? The mathematical identity may be useful when you try to find the solution $u(x,t)$,

$$e^{a+ib} = e^a(\cos b + i \sin b), \qquad a, b \text{ are real.}$$