1. (a) The ideal Hilbert Transformer is a linear time-invariant (LTI) system with frequency response given by

$$H(f) = \begin{cases} -j, & f > 0 \\ 0, & f = 0 \\ j, & f < 0 \end{cases}$$

If a signal $x(t)$ with Fourier transform given by

$$X(f) = \begin{cases} 1 - \frac{|f|}{W}, & |f| < W \\ 0, & \text{otherwise} \end{cases}$$

is passed through the ideal Hilbert transformer, and $y(t)$ is the corresponding output signal of the ideal Hilbert transformer. Find and plot the Fourier transform $Z(f)$ of the signal $z(t) = x(t) + jy(t)$. (5%)

(b) A radio frequency (RF) pulse is given by

$$g(t) = A \cdot \text{rect}(t/T) \cos(2\pi f_c t)$$

where $A > 0$ is a constant and

$$\text{rect}(t) = \begin{cases} 1, & |t| < 1/2 \\ 0, & \text{otherwise} \end{cases}$$

Find the Fourier transform $G(f)$ of $g(t)$ and sketch $|G(f)|$ assuming $f_c T \gg 1$. (10%)

(You need write down detailed derivations.)

2. Assume that $x(t)$ is a real zero-mean wide-sense stationary random process with the autocorrelation function $R_X(\tau)$ and ϕ is a random variable uniformly distributed over the interval $(0, 2\pi)$, and that $x(t)$ is statistically independent of ϕ for all t. Let

$$y(t) = x(t) \cos(2\pi f_c t + \phi)$$

(a) Show that $y(t)$ is also a wide-sense stationary random process. (5%)

(b) Find the power spectral density $S_Y(f)$ of $y(t)$. (5%)

(You need write down detailed derivations.)

3. The single sideband version of angle modulation is defined by

$$s(t) = \exp[-\hat{\phi}(t)]\cos[2\pi f_c t + \phi(t)]$$

where $\hat{\phi}(t)$ is the Hilbert transform of the phase function $\phi(t)$ and f_c is the carrier frequency. Assume single tone modulation, i.e., the phase function

$$\phi(t) = \beta \sin(2\pi f_m t)$$

where β is the modulation index and f_m is the modulation frequency.

(a) Derive the corresponding expression for the modulated wave $s(t)$. (Hint: First find the complex envelope $\tilde{s}(t)$ and then expand $\tilde{s}(t)$ into a power series by using the expansion $\exp(x) = \sum_{n=0}^{\infty} x^n/(n!)$.) (7%)

(b) Use (a) to show that the spectrum of $s(t)$ contains no frequency components in the interval $-f_c < f < f_c$. (3%)

4. Consider the noisy model of an amplitude modulation (AM) receiver shown below, with the modulated wave given by

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t)$$

where k_a is a constant, $m(t)$ is the message signal with bandwidth W and average power P, and f_c is the carrier frequency. The additive noise $w(t)$ is white Gaussian with zero mean and power spectral density $N_0/2$. The envelope detector is assumed to be ideal. Also assume that the carrier-to-noise ratio at the envelope detector input is high compared with unity.

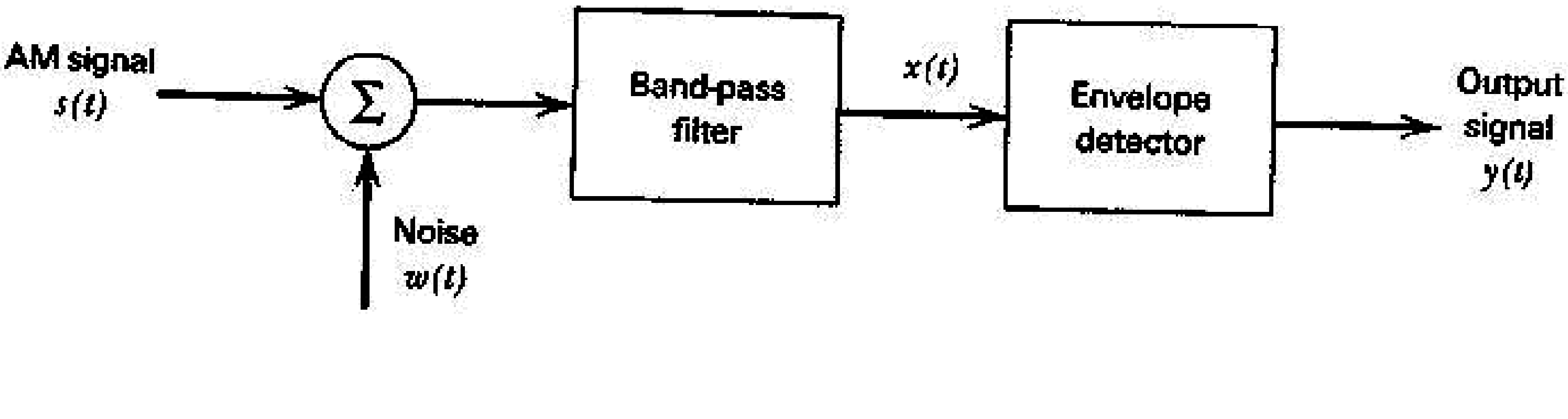
(a) Find the output signal $y(t)$. (5%)

(b) Find the output signal-to-noise ratio. (5%)

(c) Find the figure of merit of the system with single tone modulation, i.e.,

$$m(t) = A_m \cos(2\pi f_m t)$$

and 80 percent modulation. (5%)



5. One of two equally likely messages (**0** and **1**) is to be transmitted over an additive white Gaussian channel. Assume that the transmitted signal for message **0** is $s_0(t)=\phi_1(t)+\phi_2(t)$ and the transmitted signal for message **1** is $s_1(t)=-2\phi_1(t)-2\phi_2(t)$, where $\phi_1(t)$ and $\phi_2(t)$ are two orthonormal functions. Derive the maximum likelihood decision rule to detect such messages. (10%)

6. In a coherent quadriphase-shift keying (QPSK) system, the transmitted signals are defined by

$$s_i(t)=\begin{cases}\sqrt{2E/T}\cos[2\pi f_c t+(2i-1)\pi/4], & 0\le t\le T\\ 0, & \text{elsewhere}\end{cases}$$

where $i=1,2,3,4$; E is the transmitted signal energy per symbol, and T is the symbol duration; the carrier frequency f_c equals n_c/T for some fixed integer n_c. Assume that a white Gaussian noise process of zero mean and power spectral density of $N_0/2$ is added during the transmission of a QPSK signal.

(a) Find a set of orthonormal basis functions to represent this set of signals, and then draw the corresponding signal constellation. (6%)

(b) Draw the optimum receiver structure for the coherent QPSK system. (4%)

(c) Find the average probability of symbol error for the coherent QPSK system. You need to give your answer in terms of E/N_0 and the complementary error function. (10%)

(d) Determine the average probability of bit error for the coherent QPSK system with Gray encoding used for the incoming dibits (symbols) at the transmitter. (2%)

(e) What is the main advantage of a coherent QPSK system over a coherent binary phase-shift keying (BPSK) system? (3%)

國 立 清 華 大 學 命 題 紙

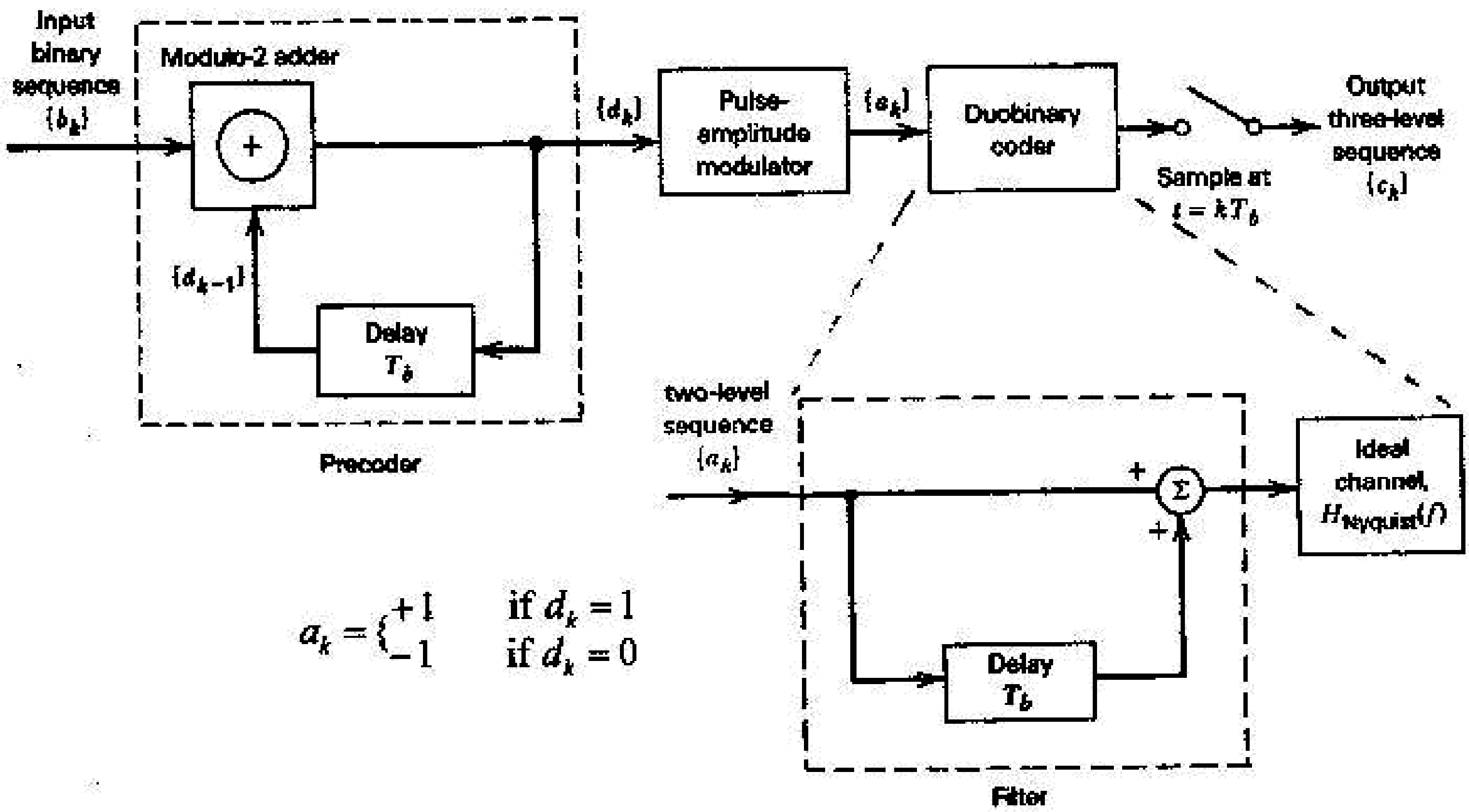
八十八學年度 通訊工程研究 系(所) 甲 組碩士班研究生招生考試

科目 通訊系統 科號 4203 共 4 頁第 4 頁 *請在試卷【答案卷】內作答

7. Consider a binary input sequence $\{b_k\}$ that consists of uncorrelated binary symbols **1** and **0**, each having duration T_b. Assume that this sequence is applied to a precoded duobinary signaling scheme as shown below.

(a) Derive a decision rule for detecting the original binary sequence $\{b_k\}$ from $\{c_k\}$. (10%)

(b) If an error is made in the detection procedure, will it propagate through the output? What will happen if the precoder is removed from the system? You have to justify your answers. (5%)



$$a_k = \begin{cases} +1 & \text{if } d_k = 1 \\ -1 & \text{if } d_k = 0 \end{cases}$$