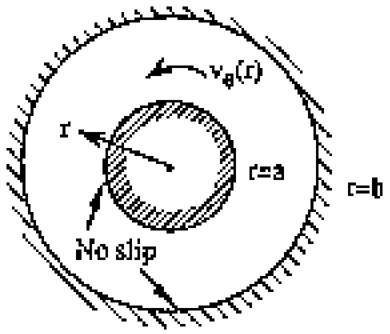
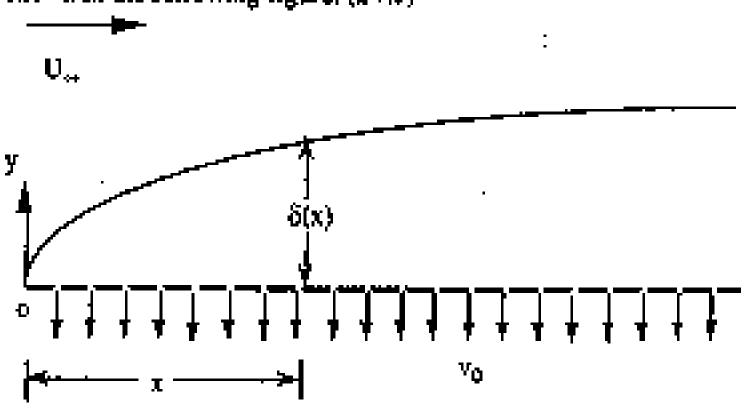
1. From the Navier-Stokes equations for incompressible flow in polar coordinates, find the most general case of purely circulating motion $v_\theta(r)$, $v_r = v_z = 0$, for flow with no slip between two fixed concentric cylinders, as in the following figure. (12%)

$$\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \nu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_\theta)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right]$$



2. Consider the steady uniform flow over a flat plate, but let the plate be porous and subject to a uniform suction $v(x,0) = -v_0$, shown in the following figure. (24%)



(a) Derive the integral momentum equation for this problem. (12%)
Hint: the equation is

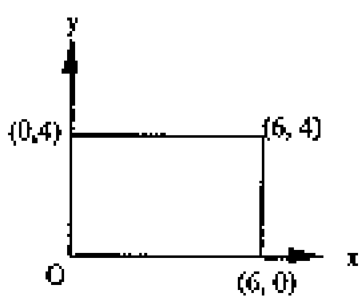
$$\frac{d}{dx}\int_0^\delta u(U_\infty - u)dy + v_0 U_\infty = \nu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

(b) Solve the integral equation by assuming (8%)

$$\frac{u}{U_\infty} = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2$$

(c) As x approaches infinity, δ = ? (4%)

3. Given a velocity $\vec{V} = x\hat{i} - y\hat{j} + 8\hat{k}$. Evaluate the circulation about a rectangular path in the xy-plane as shown in the following figure. (10%)



4. Prove that the equipotential lines are orthogonal to the streamlines. (8%)

5. Short answer (16%)

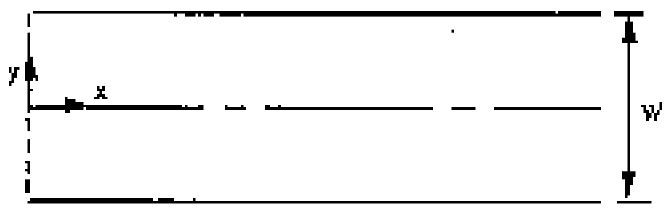
(1) What kind of body can be neutrally buoyant and remain at rest at any point where it is immersed in the fluid?

(2) What's the relationship between stream function and streamlines?

(3) Why is the method of dimensional analysis useful?

(4) There are two different kinds of fluid. How do you judge which one has the higher viscosity?

6. For a fully developed flow field between in a a parallel-plate duct as shown in the following figure. Derive the expression of the velocity distribution in the duct, i.e., the distribution of u (the velocity component in the x-direction). (15%)



$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]$$

7. A large tank of liquid has a horizontal tapered pipe of length L connected to it, as shown in the following figure. The tank is open to the atmosphere, and the surface of liquid in the tank is at a high h above the axis of the pipe. At any position x from the pipe entrance, the cross-sectional area A_x of the tapered pipe can be expressed by the following equation:

$$A_x=\frac{1}{b+cx}$$

where b and c are constants. The open e of the pipe is originally plugged. At time t = 0, the plug is pulled out of the open end, and liquid begins to flow under the action of gravity. Derive an expression relating the velocity V_e and time t after the plug is pulled out of the open end of the pipe. (15%)

Hint: for the streamline joining points O and e, the Bernoulli equation can be given by

$$\frac{p_o}{\rho}+gz_o+\frac{V_o^2}{2}=\frac{p_e}{\rho}+gz_e+\frac{V_e^2}{2}+\int_0^L\frac{\partial V}{\partial t}dx$$