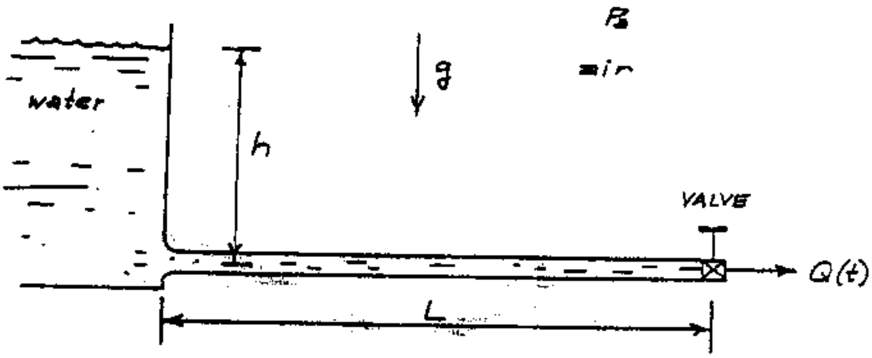
Problem 1 (25 points)



Due to insufficient rainfall so far this year, we are forced to deal with the water shortage problem in Taiwan. And here let us consider a related problem, in which water flows from a large reservoir through a very long pipe (of length L and cross-sectional area A) under a constant head h. To reduce water consumption, the local water authority has decided to shut down the water supply for a period of time (say, from 0:00 to 6:00 am) every day. When the valve is slowly closed, the head remains constant, but the volume flow rate is reduced. Also, it is agreed that friction and compressibility of water are to be neglected here.

(a) Prior to valve actuation ($t < 0$), the gage pressure just upstream of the valve is zero, and the pipe steadily supplies water at a **constant** volume flow rate of Q_0. Derive an expression for Q_0 in terms of h and other relevant physical quantities.

(b) Now let us focus our attention on the valve closure period, in which both Q and the gage pressure just upstream of the valve, p_{gv}, vary with time. Show that at any instant during the closure period, p_{gv} is given **approximately** by

$$p_{gv} = \rho\left(gh - \frac{Q^2}{2A^2} - \frac{L}{A}\frac{dQ}{dt}\right),$$

where ρ is the density of water.

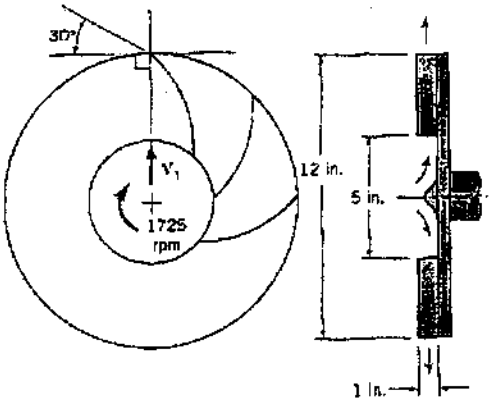
Even if you are not able to verify this result, feel free to use it in part (c) of this problem.

(c) Suppose that the "valve" is a short, frictionless nozzle with variable exit area $A_e(t)$. Prior to valve actuation ($t < 0$), a steady flow takes place with $A_e = A$ (so that $Q = Q_0$ and $P_{gv} = 0$ as mentioned in part (a) of this problem). The local water authority decided to program valve closure such that the volume flow rate decreases **linearly** from its initial steady-state value Q_0 to zero in a period τ. Show that this requires $A_e(t)$ to be programmed such that

$$\frac{A_e}{A} = \left(1 - \frac{t}{\tau}\right)\left\{1 + \frac{L}{\tau}\left(\frac{2}{gh}\right)^{1/2}\right\}^{-1/2}.$$

Problem 2 (25 points)

$Q = 6.5\ m^3/min$.



A fan has a bladed rotor of 12-in. outside diameter and 5-in. inside diameter and runs at 1725 rpm. The width of each rotor blade is 1 in. from blade inlet to outlet. The volume flowrate is steady at 6.5 m^3/min, and the absolute velocity of air at blade inlet, V_1, is purely radial. The blade discharge angle is 30° measured with respect to the tangential direction at the outside diameter of the rotor. Also, in your calculations, use $\rho_{air} = 1.23\ kg/m^3$.

(a) What would be a reasonable blade inlet angle (measured with respect to the tangential direction at the inside diameter of the rotor)?

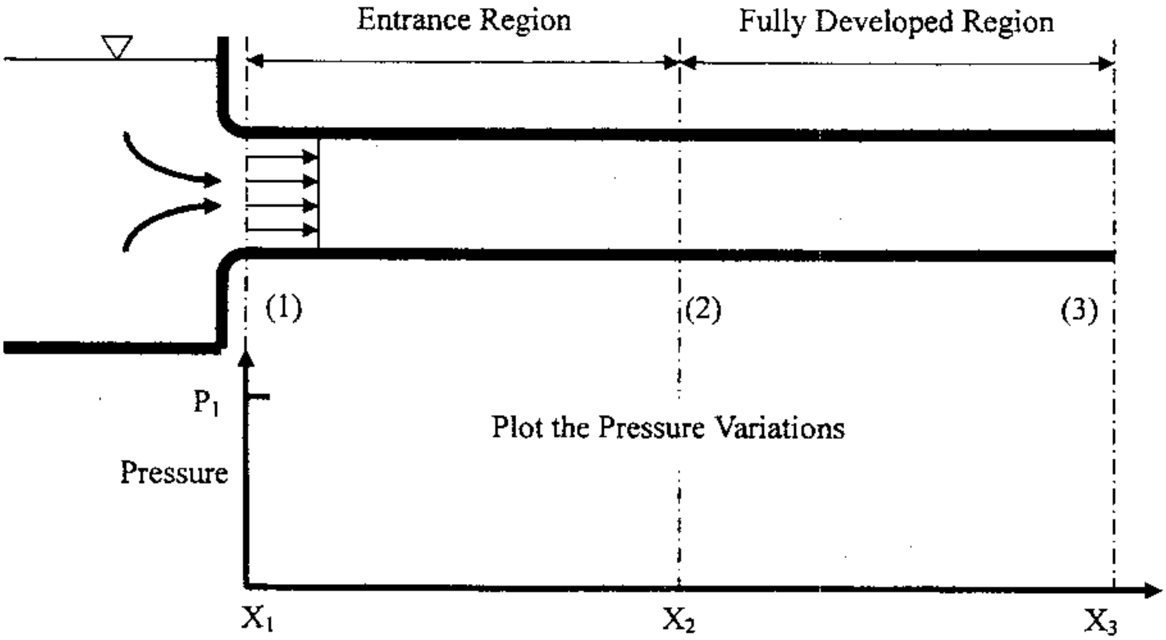
(b) Find the torque and power required to run the fan.

Problem 3 (20 points)

Considering a long horizontal pipe connected to a huge reservoir, the viscous fluid from the reservoir enters the pipe with a nearly uniform velocity profile at section (1), and continuously moves through the entrance region to finally become a fully developed flow at section (2). Please answer the following questions:

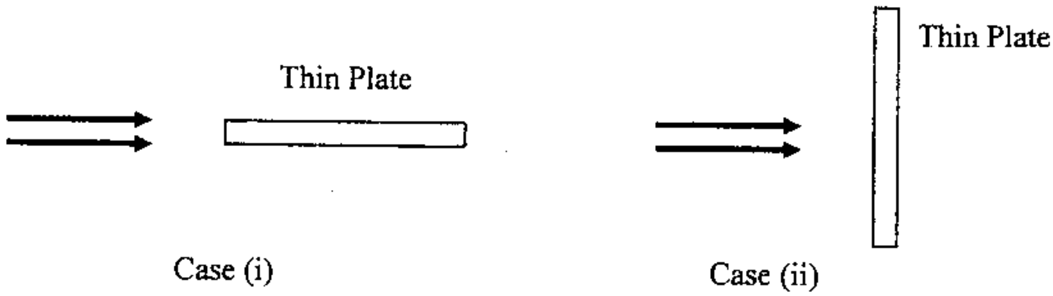
(a) Describe the flow characteristics (e.g., velocity distributions and boundary layer development) and force balances in the entrance region and the fully developed region, respectively.

(b) Plot and explain the pressure variations along the horizontal pipe from section (1) to section (3).

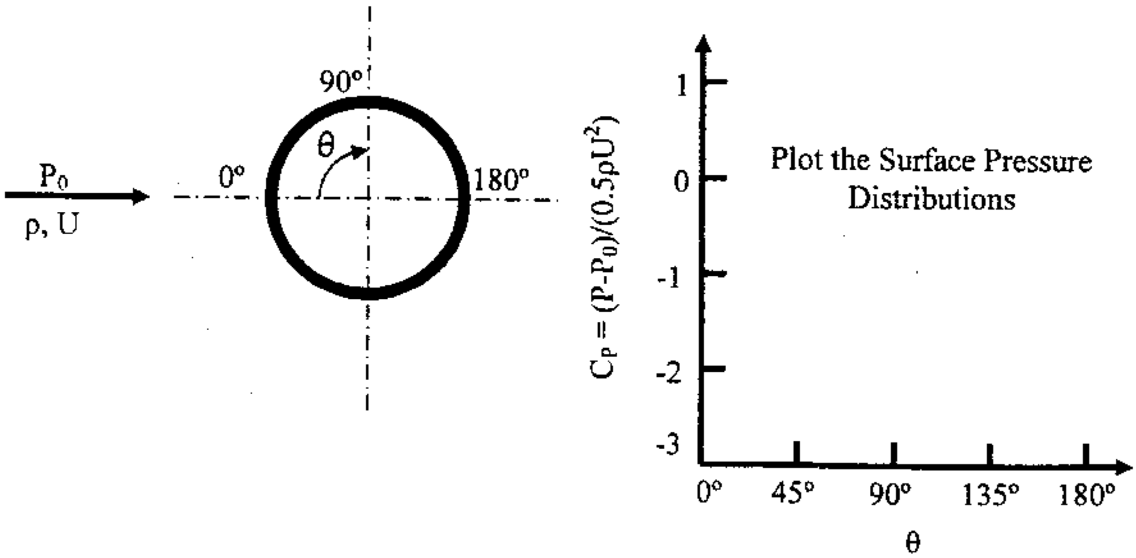


Problem 4 (30 points)

(a) A given fluid flows past a thin flat plate. In case (i) the plate is parallel to the upstream flow, and in case (ii) it is perpendicular to the upstream flow. Considering the pressure and shear stress distributions on the plate surface, describe and explain possible causes for the lift and drag on the plate for each case.



(b) A given fluid flows past a circular cylinder or sphere. Plot and discuss the surface pressure distribution for cases of an inviscid flow, a laminar boundary layer flow and a turbulent boundary layer flow, respectively.



(c) Explain the reason for dimples on golf balls.