1. (10%) The Stokes-Oseen formula for drag force F on a sphere of diameter D in a fluid stream of low velocity V, density ρ, viscosity μ, is, $F = 3\pi\mu DV + \dfrac{9\pi}{16}\rho V^2 D^2$

   Is this formula dimensionally homogeneous? (hint: use FLT or MLT system)

2. (10%) Is pressure a scalar or vector? Please prove that pressure at a point is independent of orientation

3. (20%) Consider incompressible flow in the entrance of a circular tube, as in Fig.P3. The inlet flow is uniform, $u_1 = U_0$. The flow at section 2 is developed pipe flow. Find the wall drag force F as a function of (p_1, p_2, ρ, U_0, R) if the flow at section 2 is (a) laminar:

   $u_2 = u_{max}(1 - \dfrac{r^2}{R^2})$, (b) turbulent: $u_2 = u_{max}(1 - \dfrac{r}{R})^{1/7}$

Friction drag on fluid
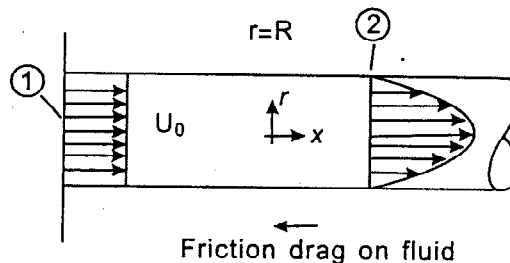
4. (20%) Consider an irrotational, incompressible, axisymmetric ($\partial/\partial\theta = 0$) flow in (r,z) coordinates. Does a stream function exist? If so, does it satisfy Laplace's equation? Does a velocity potential exist? If so, does it satisfy Laplace's equation? Are lines of constant ϕ everywhere perpendicular to the ψ lines?

5. (10%) The size of droplets produced by a liquid spray nozzle is thought to depend upon the nozzle diameter D, jet velocity U, and the properties of the liquid ρ, μ, and Y. Rewrite this relation in dimensionless form. Hint: Take D, ρ, and U as repeating variables.

6. (20%) Two immiscible fluids are contained between infinite parallel plates. The plates are separated by distance 2h, and the two fluid layers are of equal thickness. The dynamic viscosity and density of the upper fluid is half that of the lower fluid. Please find the velocity at the interface. What is the maximum velocity of the flow? Plot the velocity distribution. Both the plates are stationary. (hint: start from Navier Stokes Equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = g_x - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu}{\rho}[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}]$$

7. (10%) The Reynolds number for transition to turbulence in pipe flow was about $Re_{tr} \sim 2300$, whereas in flat plate flow $Re_{tr} \sim 1E\ 6$, nearly three orders of magnitude higher. What accounts for the difference?