EECS 2060 Discrete Mathematics Chi-chao Chao
Spring 2021

Homework Assignment No. 1
Due 10:10am, March 17, 2021

Reading: Biggs: Chapters 1 Statements and Proofs, 3 The Logical Framework; Grimaldi: Sections 3.1 Sets and Subsets, 3,2 Set Operations and the Laws of Set Theory, 4.1 The Well-Ordering Principle: Mathematical Induction, 4.2 Recursive Definitions, 5.2 Functions: Plain and One-to-One, 5.3 Onto Functions: Stirling Numbers of the Second Kind (up to right before Example 22), 5.4 Special Functions, 5.5 The Pigeonhole Principle, 5.6 Function Composition and Inverse Functions.

Problems for Solution:

1. Determine whether or not the given pairs are logically equivalent, where p, q, and r are statements.
   (a) $\neg(p \Leftrightarrow q)$ and $\neg p \Leftrightarrow q$.
   (b) $(p \wedge q) \Rightarrow r$ and $(p \Rightarrow r) \vee (q \Rightarrow r)$.
   (c) $p \Rightarrow (q \vee r)$ and $(p \Rightarrow q) \wedge (p \Rightarrow r)$.

2. The *symmetric difference* of sets A and B, denoted by $A \bigtriangleup B$, is defined to be the set containing those elements in either A or B, but not in both A and B, i.e., $A \bigtriangleup B = (A \cup B) - (A \cap B)$.
   (a) Find the symmetric difference of $\{1, 2, 3, 4, 5\}$ and $\{4, 5, 6, 7, 8\}$.
   (b) Show that $A \bigtriangleup B = (A - B) \cup (B - A)$.

3. The nth *Fibonacci number* F_n is defined recursively by

$$F_0 = 0, \quad F_1 = 1, \quad F_{n+1} = F_n + F_{n-1}, \text{ for } n \geq 1.$$

   Show that

$$\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}, \quad \text{for all } n \geq 1.$$

4. What is wrong with the following "proof?"

   Theorem $a^n = 1$ for all nonnegative integers n, whenever a is a nonzero real number.

   Proof *Induction basis:* $a^0 = 1$ is true by the definition of a^0.

   Induction step: Assume that $a^j = 1$ for $j = 0, 1, \ldots, k$. We then have

$$a^{k+1} = \frac{a^k \cdot a^k}{a^{k-1}} = \frac{1 \cdot 1}{1} = 1$$

   which completes the induction step. ∎

5. Consider functions $f : A \to B$ and $g : B \to C$. For each of the following statements, prove it if it is true; otherwise, find a counterexample.

   (a) If $g \circ f$ is surjective, then so is f.

   (b) If $g \circ f$ is surjective, then so is g.

6. The function $f : A \to B$ is said to have a *left inverse* $l : B \to A$ if

$$(l \circ f)(a) = a, \text{ for all } a \in A.$$

   (a) Show that if f has a left inverse then it is injective.

   (b) Show that if f is injective then it has a left inverse.

7. Prove that if any 101 different numbers are selected from the set $\{1, 2, 3, \ldots, 200\}$, then two of the chosen numbers are consecutive.

8. Prove that if $S = \{1, 2, 3, \ldots, 2n + 1\}$, for $n \in \mathcal{N}$, then any subset of size $n + 2$ from S must contain two elements whose sum is $2n + 2$.

Homework Collaboration Policy: I allow and encourage discussion or collaboration on the homework. However, you are expected to write up your own solution and understand what you turn in. Late homework is subject to a penalty of 5% to 40% of your total points.