Solution to Homework Assignment No. 5

1. (a) No. From the corresponding undirected graph shown in Fig. 1, we can see that G is connected but not every vertex of G, e.g., D and E, has even degree.

(b) Yes. An Euler trail is given as

$$D, C, A, C, F, A, F, B, F, E, B, A, E, A, D, E.$$

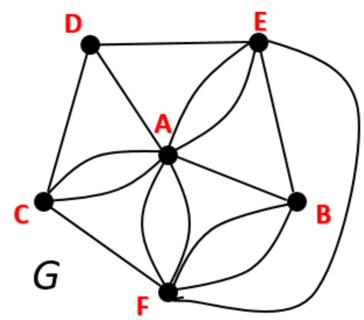


Figure 1: Graph G for Problem 1.

2. (a) A planar embedding of the graph is shown in Fig. 2, so this graph is planar.

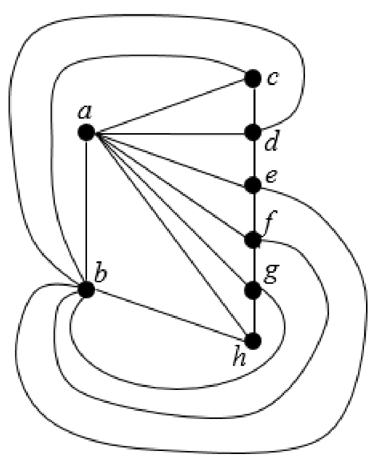


Figure 2: A planar embedding of the graph in Problem 2.(a).

(b) This graph contains a subgraph that is homeomorphic to K_5 as shown in Fig. 3, so it is nonplanar.

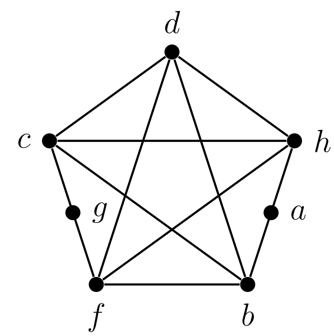


Figure 3: A subgraph of the graph in Problem 2.(b).

3. (a) If this graph has no cycles, since $e > 2$, the number of edges must be at least 3. If there are cycles, since it is a simple graph, the length of the shortest cycle is at least 3. Hence the girth g is at least 3. For any connected planar simple graph with $g \geq 3$, the edge-vertex inequality gives

$$e \leq (g/(g-2))(v-2).$$

Since $g \geq 3$, $g \leq 3g - 6$, which implies $g/(g-2) \leq 3$. Therefore,

$$e \leq 3(v-2) = 3v - 6.$$

(b) If all vertices have degree at least 6, then

$$e \geq 6v/2 = 3v$$

which contradicts $e \leq 3v - 6$. Therefore, G is nonplanar.

4. A Hamilton cycle is given as

$$a, b, c, d, e, j, i, h, g, l, m, n, o, t, s, r, q, p, k, f, a.$$

5. (a) This can be proved by contrapositive. Assume that the graph G has no cycles. Then G is a tree since G is connected. However, G has the same numbers of vertices and edges, which violates the property $|V| = |E| + 1$. Hence, G has at least one cycle.

(b) Let i be the number of internal vertices. Then $n = mi + 1$. We have

$$\begin{aligned} l &= n - i \\ &= n - \frac{n-1}{m} \\ &= \frac{(m-1)n+1}{m}. \end{aligned}$$

6. Preorder traversal: $a, b, d, e, i, m, n, o, j, c, f, g, h, k, p, l$.
Postorder traversal: $d, m, n, o, i, j, e, b, f, g, p, k, l, h, c, a$.
Inorder traversal: $d, b, m, i, n, o, e, j, a, f, c, g, p, k, h, l$.

7. (a) The depth-first spanning tree is shown in Fig. 4. The height of the tree is 9.

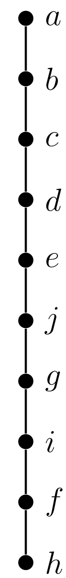


Figure 4: Depth-first spanning tree for Problem 7.(a).

(b) The breadth-first spanning tree is shown in Fig. 5. The height of the tree is 2.

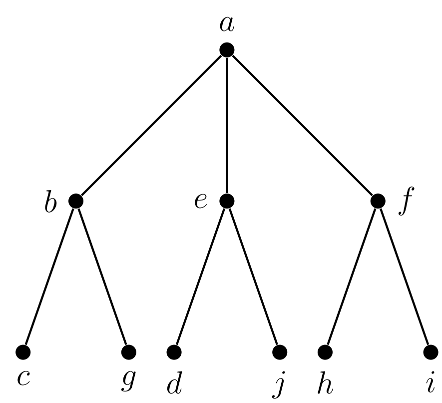
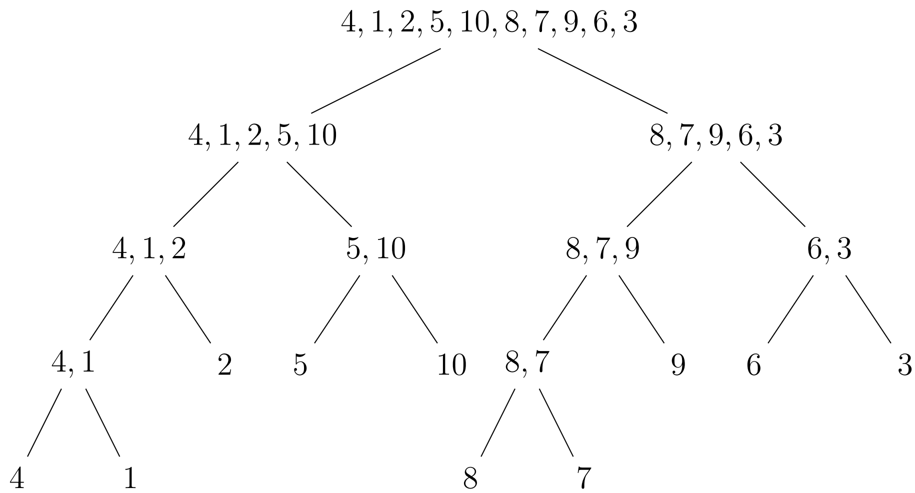
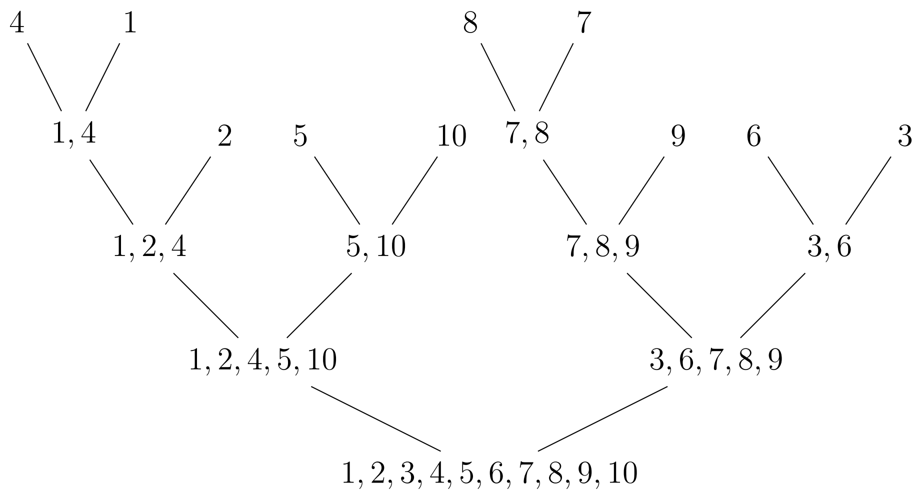


Figure 5: Breadth-first spanning tree for Problem 7.(b).

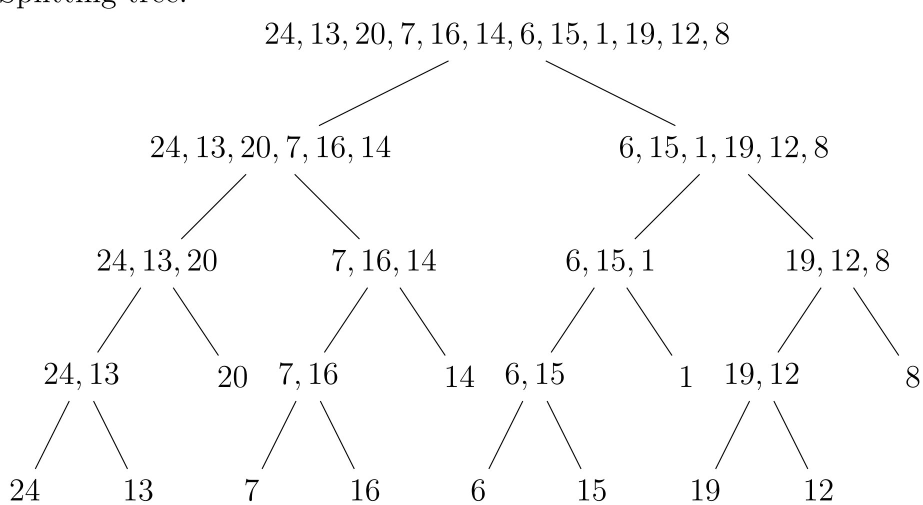
8. (a) Splitting tree:



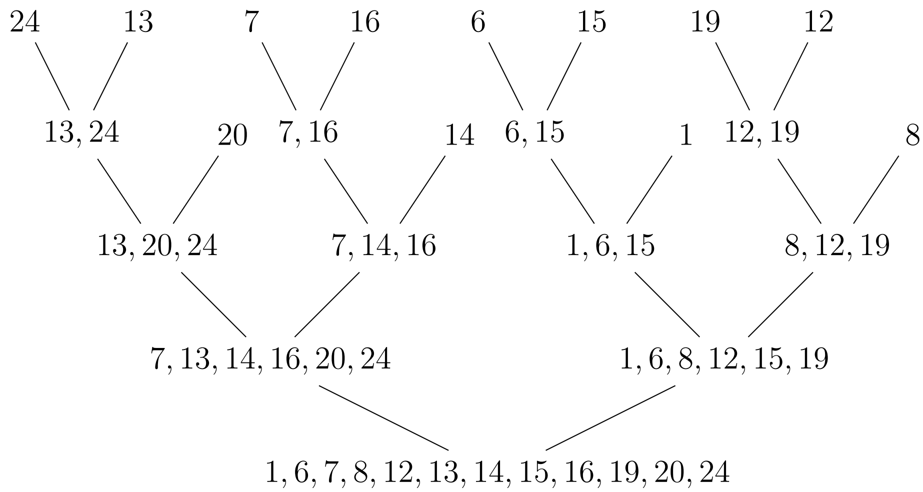
Merging tree:



(b) Splitting tree:



Merging tree: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