微積分一　題庫

【章節 2.1】

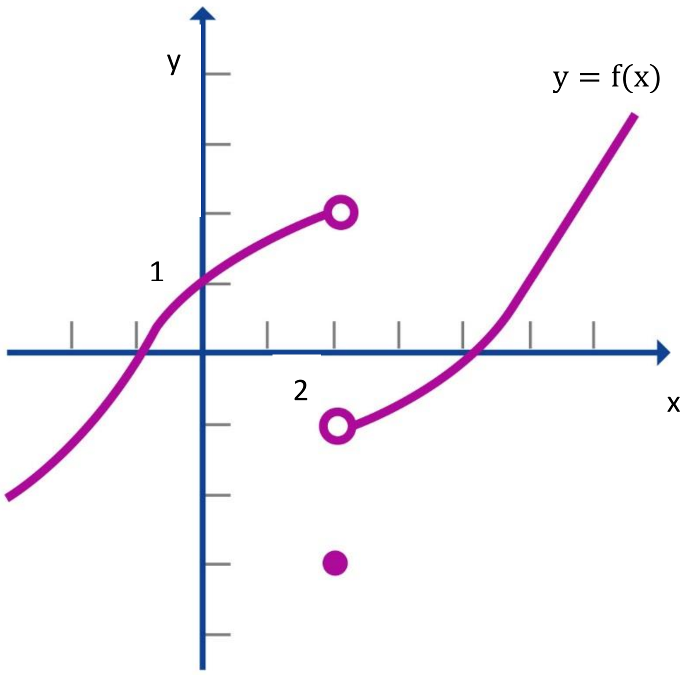
p.61 (1、2、3、5、11. Determine the limit by drawing the graph：41、45、46)

Exercises1-10. You are given a number *c* and the graph of function *f.*

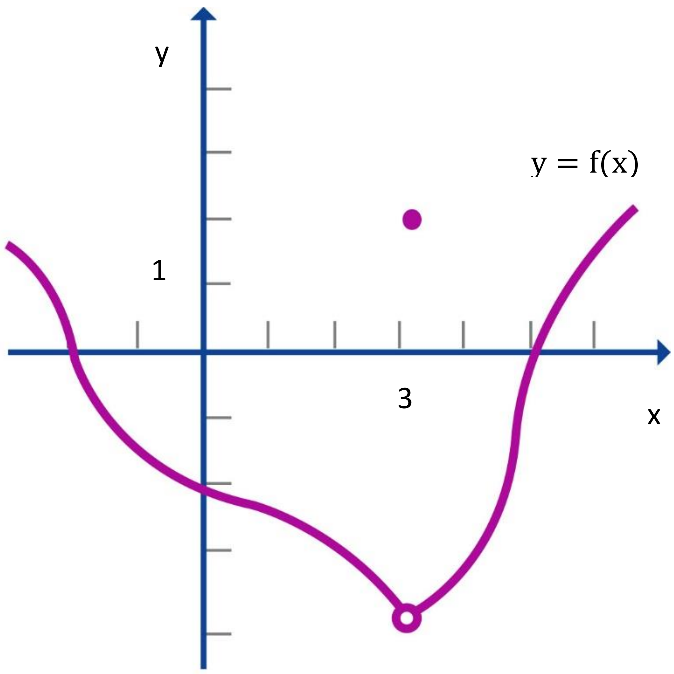
Use the graph to find

(a) $\lim\limits_{x\to c-} f(x)$ (b) $\lim\limits_{x\to c+} f(x)$ (c) $\lim\limits_{x\to c} f(x)$ (d) $f(c)$

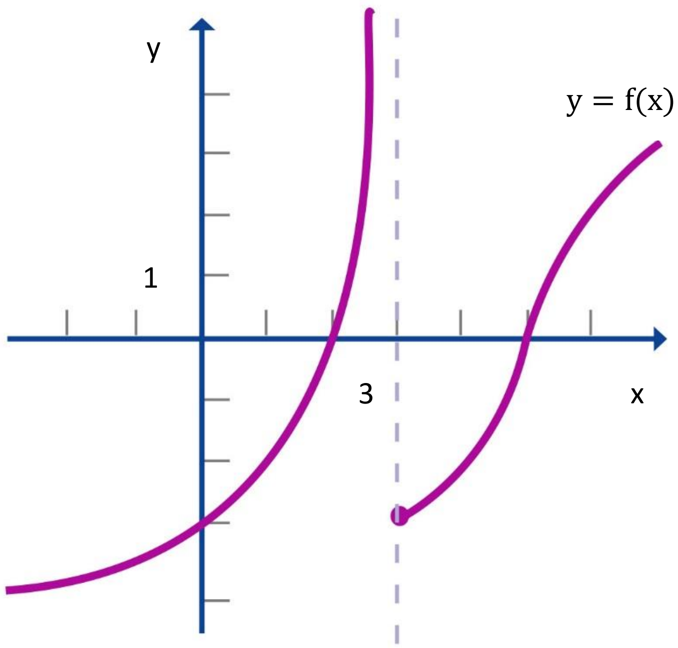
1. c = 2.



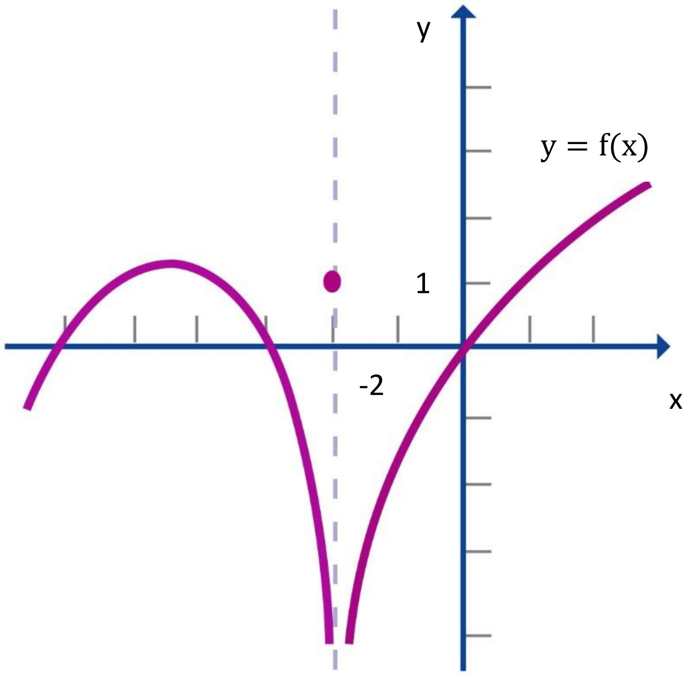
2. c = 3.



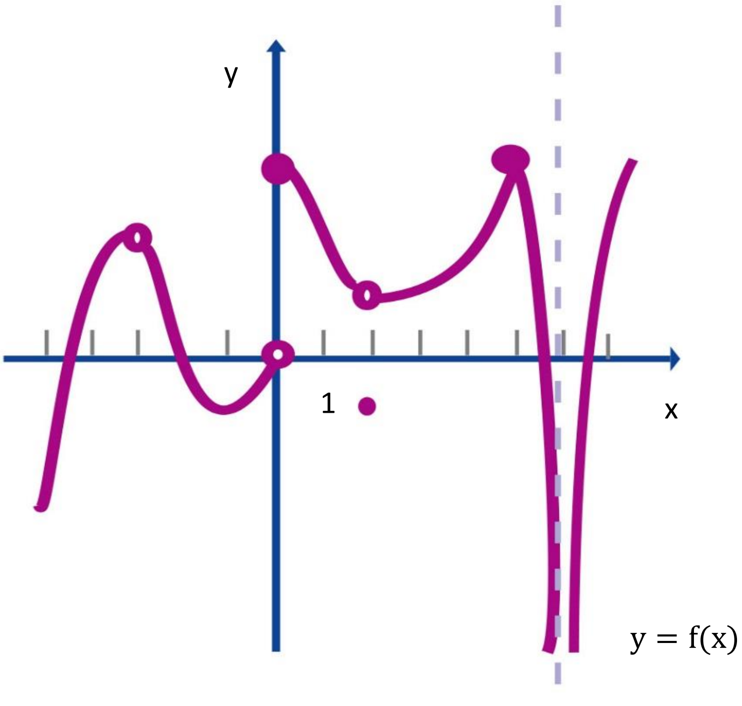
3. c = 3.



5. c = -2.



11. Give the values of c for which $\lim_{x \to c} f(x)$ does not exist.



41. Determine the limit by drawing the graph: $\lim_{x \to 0} f(x)$; $f(x) = \begin{cases} x^2, & x < 0 \\ 1 + x, & x > 0. \end{cases}$

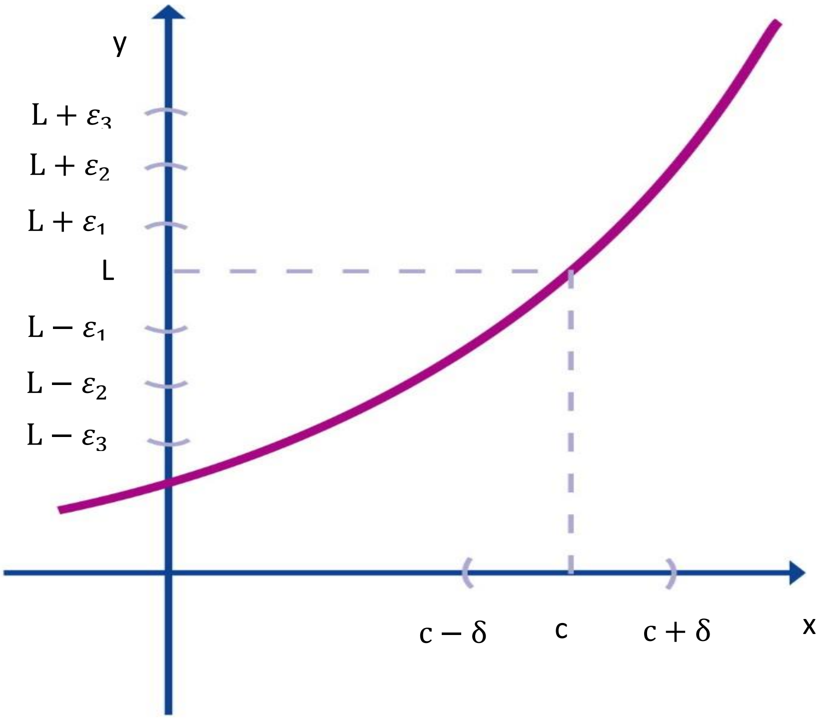
45. Determine the limit by drawing the graph: $\lim_{x \to 0} f(x)$; $f(x) = \begin{cases} 2, & x \text{ rational} \\ -2, & x \text{ irrational}. \end{cases}$

46. Determine the limit by drawing the graph: $\lim_{x \to 1} f(x)$; $f(x) = \begin{cases} 2x, & x \text{ rational} \\ 2, & x \text{ irrational}. \end{cases}$

【章節 2.2】

p.71 (22、26、27、35、39、42、45、51、53、54、62)

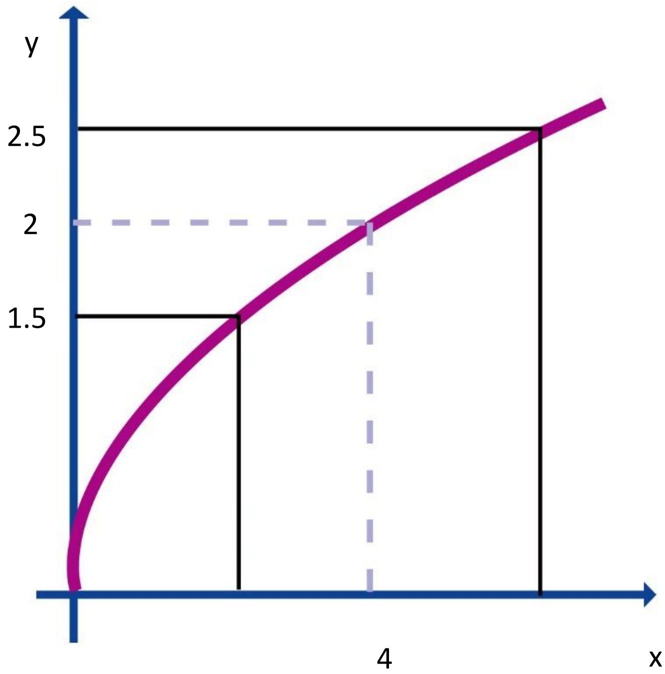
22. For which of the ε 's given in the figure does the specified δ work?



Exercises 23-26. Find the largest δ that "works" tor the given ε .

26. $\lim_{x\to 2}\left(\frac{1}{5}x\right) = \frac{2}{5}$; $\varepsilon = 0.1$.

27. The graphs of $f(x) = \sqrt{x}$ and the horizontal lines y=1.5 and y=2.5 are shown in the figure. Use a graphing utility to find a δ >0 which is such that if $0 < |x-4| < \delta$, then $\left|\sqrt{x}-2\right| < 0.5$.



35. Give an ε, δ proof for the following statements. $\lim_{x\to 4}(2x-5)=3.$

39. Give an ε, δ proof for the following statements. $\lim_{x\to 2}|1-3x|=5.$

42. Suppose that $|A-B|<\varepsilon$ for each $\varepsilon>0$. Prove that A=B. HINT: Suppose that A$\neq$B and set $\varepsilon=\frac{1}{2}|A-B|$.

45. Proof that

$$\lim_{x\to c} f(x)=0, \quad iff \quad \lim_{x\to c}|f(x)|=0.$$

(2.2.10)

51. Give an ε, δ proof for the following statements. $\lim_{x\to 1} x^3=1.$

53. Give an ε, δ proof for the following statements. $\lim_{x\to 3^-}\sqrt{3-x}=0.$

54. Prove that, for the function $g(x)=\begin{cases} x, & x\ rational \\ 0, & x\ irrational, \end{cases}$ $\lim_{x\to 0} g(x)=0.$

62. Prove that if $\lim_{x\to c} f(x)=L$, then there are positive numbers δ and B such that if $0<|x-c|<\delta$, then $|f(x)|<B$.

【章節 2.3】

p.79 (6、21、33、34、38、42~52、55)

Exercises 5-38. Evaluate the limits that exist.

6. $\lim_{x\to 3}(5-4x)^2$.

21. $\lim_{x\to 4}\left(\frac{\sqrt{x}-2}{x-4}\right)$.

33. $\lim_{x\to 1}\left(\frac{x^5-1}{x^4-1}\right)$.

34. $\lim_{h\to 0}h^2\left(1+\frac{1}{h}\right)$.

38. $\lim_{x\to -4}\left(\frac{2x}{x+4}-\frac{8}{x+4}\right)$.

42. Give that $f(x)=x^3$, evaluate the limits that exist.

(a) $\lim_{x\to 3}\frac{f(x)-f(3)}{x-3}$.

(b) $\lim_{x\to 3}\frac{f(x)-f(2)}{x-3}$.

(c) $\lim_{x\to 3}\frac{f(x)-f(3)}{x-2}$.

(d) $\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$.

43. Show by example that $\lim_{x\to c}[f(x)+g(x)]$ can exist even if $\lim_{x\to c}f(x)$ and $\lim_{x\to c}g(x)$ do not exist.

44. Show by example that $\lim_{x\to c}[f(x)g(x)]$ can exist even if $\lim_{x\to c}f(x)$ and $\lim_{x\to c}g(x)$ do not exist.

45. True or false? Justify your answers. If $\lim_{x\to c}[f(x)+g(x)]$ exists but $\lim_{x\to c}f(x)$ does not exist, then $\lim_{x\to c}g(x)$ does not exist.

46. True or false? Justify your answers. If $\lim_{x\to c}[f(x)+g(x)]$ and $\lim_{x\to c}f(x)$ exist, then it can happen that $\lim_{x\to c}g(x)$ does not exist.

47. True or false? Justify your answers. If $\lim_{x\to c}\sqrt{f(x)}$ exists, then $\lim_{x\to c} f(x)$ exists.

48. True or false? Justify your answers. If $\lim_{x\to c} f(x)$ exists, then $\lim_{x\to c}\sqrt{f(x)}$ exists.

49. True or false? Justify your answers. If $\lim_{x\to c} f(x)$ exists, then $\lim_{x\to c}\frac{1}{f(x)}$ exists.

50. True or false? Justify your answers. If $f(x) \le g(x)$ for all $x \neq c$, then $\lim_{x\to c} f(x) \le \lim_{x\to c} g(x)$.

51. True or false? Justify your answers. If $f(x) < g(x)$ for all $x \neq c$, then $\lim_{x\to c} f(x) < \lim_{x\to c} g(x)$.

52. (a) Verify that

$$\max\{f(x), g(x)\} = \frac{1}{2}\{[f(x) + g(x)] + |f(x) - g(x)|\}.$$

(b) Find a similar expression for min{f(x), g(x)}.

55. (a) Suppose that $\lim_{x\to c} f(x) = 0$ and $\lim_{x\to c}[f(x)g(x)] = 1$. Prove that $\lim_{x\to c} g(x)$ does not exist.

(b) Suppose that $\lim_{x\to c} f(x) = L \neq 0$ and $\lim_{x\to c}[f(x)g(x)] = 1$. Does $\lim_{x\to c} g(x)$ exist, and if so, what is it?

※補充題：Let $\lim_{x\to c} f(x) = 5$ and $\lim_{x\to c} g(x) = 1$. Using $\varepsilon - \delta$ argument to prove that 1$\lim_{x\to c}[3f(x) - g(x)] = 14$, 2$\lim_{x\to c}[2\, f(x)g(x)] = 10$.

【章節 2.4】

p.88 (35、37、52、53、54、55)

35. Let $f(x)=\begin{cases} x^2, & x<1 \\ Ax-3, & x\geq 1. \end{cases}$ Find A given that f is continuous at 1.

37. Give necessary and sufficient condition on A and B for the function

$$f(x)=\begin{cases} Ax-B, & x\leq 1 \\ 3x, & 1<x<2 \\ Bx^2-A, & 2\leq x \end{cases}$$

to be continuous at x=1 but discontinuous at x=2.

52. (a) Prove that if f is continuous everywhere, then $|f|$ is continuous everywhere.

(b) Give an example to show that the continuity of $|f|$ does not imply the continuity of f.

(c) Give an example of a function f such that f is continuous nowhere, but $|f|$ is continuous everywhere.

53. Suppose the function f has the property that there exists a number B such that

$$|f(x)-f(c)|\leq B|x-c|$$

for all x in the interval (c-p, c+p). Prove that f is continuous at c.

54. Suppose the function f has the property that

$$|f(x)-f(t)|\leq |x-t|$$

for each pair of points x, t in the interval (a, b). Prove that f is continuous on (a, b).

55. Prove that if

$$\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

exists, then f is continuous at c.

※補充題：Show that f(x) = $\sqrt[3]{x^2+2x}+\frac{|4x+5|}{x^2-2x+1}$ is continuous everywhere except at x=1.

【章節 2.5】

p.96 (6、12、18、43、46、47、49、50)

Exercises 1-32. Evaluate the limits that exist.

6. $\lim_{x\to 0}\left(\frac{\sin 3x}{5x}\right)$.

12. $\lim_{x\to 0}\left(\frac{\tan^2 3x}{4x^2}\right)$.

18. $\lim_{x\to 0}\left(\frac{x^2-2x}{\sin 3x}\right)$.

43. Show that $\lim_{x\to 0} x\sin(1/x) = 0$. HINT: Use the pinching theorem.

46. Let f be the Dirichlet function

$$f(x) = \begin{cases} 1, & x\ rational \\ 0, & x\ irrational. \end{cases}$$

Show that $\lim_{x\to 0} xf(x) = 0$.

47. Prove that if there is a number B such that $|f(x)| \leq B$ for all x$\neq$0, then $\lim_{x\to 0} xf(x) = 0$.

NOTE: Exercises 43-46 are special cases of this general result.

49. Prove that if there is a number B such that $|f(x) - L|/|x - c| \leq B$ for all x$\neq$c, then $\lim_{x\to c} f(x) = L$.

50. Given that $\lim_{x\to c} f(x) = 0$ and $|g(x)| \leq B$ for all x in an interval (c-p, c+p), prove that $\lim_{x\to c} f(x)g(x) = 0$.

【章節 2.6】

p.100 (3、11、26、28、29)

Exercises 1-8. Use the intermediate-value theorem to show that there is a solution of the given equation in the indicated interval.

3. $\sin x + 2\cos x - x^2 = 0;\quad [0,\pi/2]$.

11. Show that the equation $x^3 - 4x + 2 = 0$ has three distinct roots in [-3,3] and locate the roots between consecutive integers.

26. Given that f and g are continuous on [a, b], that f(a)<g(a), and g(b)<f(b), show that there exist at least one number c in (a, b) such that f(c)=g(c). HINT: consider f(x)-g(x).

28. Use the intermediate-value theorem to prove that every real number has a cube root. That is, prove that for any real number a there exists a number c such that $c^3 = a$.

29. The intermediate-value theorem can be used to prove that each polynomial equation of odd degree $x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0$ with n odd has at least one real root. Show that the cubic equation $x^3 + ax^2 + bx + c = 0$ has at least one real root.

※補充題：Let f be continuous on [a,b]. if $-\sqrt{3}, \frac{2}{3} \in f([a,b])$, then

$[-\sqrt{3}, \frac{2}{3}] \subset f([a,b])$.

【章節 3.1】

p.112 (6、19、32、40、49、59)

Exercises 1-10. Differentiate the function by forming the difference quotient

$$\frac{f(x+h)-f(x)}{h}$$

And taking the limit as h tends to 0.

6. f(x) = 1/(x+3).

Exercises 17-20. Write an equation for the tangent line at (c,f(c)).

19. f(x) = $1/x^2$; c = -2.

Exercises 29-32 find f'(c) if it exist.

32. f(x) = $\begin{cases} -\frac{1}{2}x^2, x<3 \\ -3x, \ \ x \geq 3 \end{cases}$; c = 3.

40. Set $\mathrm{f(x)} = \begin{cases} (x+1)^2, & x \leq 0 \\ (x-1)^2, & x > 0. \end{cases}$

(a) Determine f'(x) for x≠0.

(b) Show that f is not differentiable at x=0.

49. Set $\mathrm{f(x)} = \begin{cases} x^2-2, & x \leq 2 \\ 2x-2, & x > 2. \end{cases}$

(a) Show that f is continuous at 2.

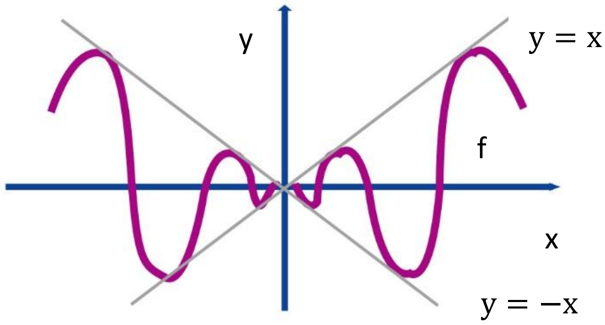
(b) Is f differentiable at 2?

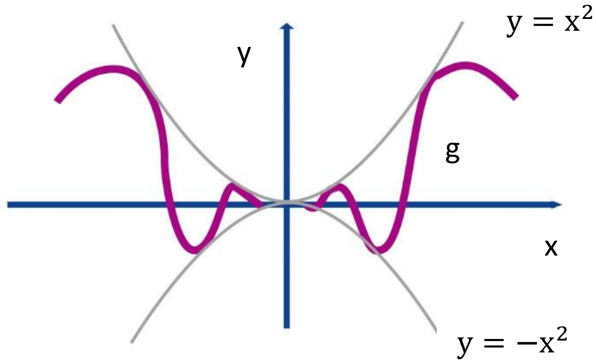
59. Let $\mathrm{f(x)} = \begin{cases} x\sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$ and g(x)=xf(x). The graphs of f and g are indicated in the figures below.

(a) Show that f and g are both continuous at 0.

(b) Show that f in not differentiable at 0.

(c) Show that g is differentiable at 0 and give g'(0).





【章節 3.2】

p.122 (9、14、20、28、30、66)

Exercises 1-20. Differentiate

9. G(x) = $(x^2 - 1)(x - 3)$.

14. $G(x) = \frac{7x^4+11}{x+1}$.

20. $G(x) = (1 + \frac{1}{x})(1 + \frac{1}{x^2})$.

28. Find f’(0) given that h(0)=3 and h’(0)=2. f(x)=$3x^2h(x) - 5x$.

30. Find f’(0) given that h(0)=3 and h’(0)=2. f(x)=$h(x) + \frac{x}{h(x)}$.

66. Verify that, if f, g, h are differentiable, then $(fgh)'(x) = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$. HINT: Apply the product rule to [f(x)g(x)]h(x).

【章節 3.3】

p.128 (14、20、26、38、56)

Exercises 11-22. Find the indicates derivative.

14. $\frac{d}{dx}[(2x^2+3x^{-1})(2x-3x^{-2})]$.

20. $\frac{d}{du}[u^2(1-u^2)(1-u^3)]$.

Exercises 23-26. Evaluate dy/dx at x=2.

26. y = $\frac{(x^2+1)(x^2-2)}{x^2+2}$.

Exercises 33-38. Find d^3y/dx^3.

38. y = $\frac{x^4+2}{x}$.

56. Verify the identity $\mathrm{f(x)g''(x)-f''(x)g(x)} = \frac{d}{dx}[f(x)g'(x)-f'(x)g(x)]$.

【章節 3.4】

p.132 (6、7)

6. Find the values of x at which the rate of change of y = $x^3-12x^2+45x-1$ with respect to x is zero.
7. Find the rate or change of the volume of a sphere with respect to the radius r.

【章節 3.5】

p.138 (5、7、16、24、27、44、45、60)

Exercises 1-6. Differentiate the function: (a) by expanding before differentiation, (b) by using the chain rule. Then reconcile your results.

5. y = $(\mathrm{x}+x^{-1})^2$.

Exercises 7-20. Differentiate the function.

7. f(x) = $(1-2\mathrm{x})^{-1}$.

16. f(x) = $(\frac{4x+3}{5x-2})^3$.

Exercises 21-24. Find dy/dx at x = 0.

24. y = u^3-u+1, u = $\frac{1-x}{1+x}$.

Exercises 27-28. Find dy/dx at x = 2.

27. y = $(s+3)^2$, s = $\sqrt{t-3}$, $t = x^2$.

44. Express the derivative in prime notation. $\frac{d}{dx}[f(\frac{x-1}{x+1})]$.

45. Express the derivative in prime notation. $\frac{d}{dx}[[f(x)]^2+1]$.

60. Let f and g be differentiable functions such that f'(x)=g(x) and g'(x)=f(x), and let $\mathrm{H(x)} = [f(x)]^2 - [g(x)]^2$. Find H'(x).

【章節 3.6】

p.145 (12、24、27、55、56、67)

Exercises 1-12. Differentiate the function.

12. y = $[x^2 - \sec 2x]^3$.

Exercises 13-24. Find the second derivative.

24. y= $\sec^2$x−$\tan^2$x.

Exercises 25-30. Find the indicated derivative.

27. $\frac{d}{dt}[t^2\frac{d^2}{dt^2}(t\cos 3t)]$.

55. It can be shown by induction that the nth derivative of the sine function is given by the formula

$$\frac{d^n}{dx^n}(\sin x) = \begin{cases} (-1)^{(n-1)/2}\cos x, & n\ odd \\ (-1)^{n/2}\sin x, & n\ even. \end{cases}$$

Persuade yourself that this formula is correct and obtain a similar formula for the nth derivative of the cosine function.

56. Verify the following differentiation formulas:

(a) $\frac{d}{dx}(\cot x) = -csc^2x.$

(b) $\frac{d}{dx}(\sec x) = \sec x\tan x.$

(c) $\frac{d}{dx}(\csc x) = -\csc x\cot x.$

67. Set $\mathrm{f(x)} = \begin{cases} x\sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$ and g(x)=xf(x). In Exercise 62, Section 3.1, you were asked to show that f is continuous at 0 but not differentiable there, and that g is differentiable at 0. Both f and g are differentiable at each x≠0.

(a) Find f'(x) and g'(x) for x≠0.

(b) Show that g' is not continuous at 0.

【章節 3.7】

p.150 (10、18、32、34、42、48)

Exercises 1-10. Use implicit differentiation to express dy/dx in terms of x and y.

10. $\tan xy = xy$.

18. Evaluate dy/dx and d^2y/dx^2 at the point indicated. $x^2 + 4xy + y^3 + 5 = 0$; (2,-1).

32. Find dy/dx. $y = \sqrt{(x^4 - x + 1)^3}$.

34. Carry out the differentiation. $\frac{d}{dx}\left(\sqrt{\frac{3x+1}{2x+5}}\right)$.

42. Find the second derivative. $y = \sqrt{x}\sin\sqrt{x}$.

48. Find the angles at which the circles $(x-1)^2 + y^2 = 10$ and $x^2 + (y-2)^2 = 5$ intersect.

【章節 4.1】

p.158 (4、9、12、23、25、26、29、35、39、40、42)

Exercises 1-4. Show that f satisfies the conditions of Rolle's theorem on the indicated interval and find all numbers c on the interval for which f'(c) = 0.

4. f(x) = $x^{2/3} - 2x^{1/3}$; [0,8].

9. Verify that f satisfies the conditions of the mean-value theorem on the indicated interval and find all numbers c that satisfy the conclusion of the theorem. $f(x) = \sqrt{1-x^2}$; [0, 1].

12. The function $f(x) = x^{2/3} - 1$ has zeros at x=-1 and at x=1.

(a) Show that f' has no zeros in (-1, 1).

(b) Show that this does not contradict Rolle's theorem.

23. Show that the equation $6x^4 - 7x + 1 = 0$ does not have more than two distinct real roots.

25. Show that the equation $x^3 + 9x^2 + 33x - 8 = 0$ has exactly one real root.

26. (a) Let f be differentiable on (a, b). Prove that if f'(x)≠0 for each x∈(a, b), then f has at most one zero in (a, b).

(b) Let f be twice differentiable on (a, b). Prove that if f''(x)≠0 for each x∈(a, b), then f has at most two zeros in (a, b).

29. A number c is called a fixed point of f if f(c)=c. Prove that if f is differentiable on an interval I and f'(x)<1 for all x∈I, then f has at most one fixed point in I. HINT: form g(x)=f(x)-x.

35. Given that $|f'(x)| \leq 1$ for all real numbers x, show that $|f(x_1) - f(x_2)| \leq |x_1 - x_2|$ for all real numbers x_1 and x_2.

39. Let f be differentiable on (a, b) and continuous on [a, b].

(a) Prove that if there is a constant M such that f'(x)≦M for all x∈(a, b), then f(b)≦f(a)+M(b-a).

(b) Prove that if there is a constant m such that f'(x)≧m for all x∈(a, b), then f(b)≧f(a)+m(b-a).

(c) Parts (a) and (b) together imply that if there exists a constant K such that |f'(x)|≦K on (a, b), then f(a)-K(b-a)≦f(b)≦f(a)+K(b-a).

40. Suppose that f and g are differentiable functions and f(x)g'(x)-g(x)f'(x) has no zeros on some interval I. Assume that there are numbers a, b in I with a<b for which f(a)=f(b)=0, and that f has no zeros in (a, b). Prove that if g(a)≠0 and g(b)≠0, then g has exactly one zero in (a, b). HINT: Suppose that g has no zeros in (a, b) and consider h=f/g. Then consider k=g/f.

42. *(Important)* Use the mean-value theorem to show that if f is continuous at x and at x+h and is differentiable between these two numbers, then

$$f(x+h) - f(x) = f'(x+\theta h)h$$

for some number θ between 0 and 1. (In some texts this is how the mean-value theorem is stated.)

※補充題：Let f be differentiable on [a,b]. if f'(a)>0 and f'(b)<0 , then there exists c∈(a,b) such that f'(c) = 0. (Do not assume that f' is continuous.)

【章節 4.2】

p.165 (15、24、30、55、56、58)

Exercises 1-24. Find the intervals on which f increases and the intervals on which f decreases.

15. f(x) = $\frac{x-1}{x+1}$.

24. f(x) = $\sin^2 x - \sqrt{3}\sin x, \ 0 \leq x \leq \pi$.

30. Define f on the domain indicated given the following information.

$(0,\infty); \ f'(x) = x^{-5} - 5x^{-1/5}; \ f(1) = 0.$

55. Suppose that for all real x f'(x)=-g(x) and g'(x)=f(x).

(a) Show that $f^2(x) + g^2(x) = C$ for some constant C.

(b) Suppose that f(0)=0 and g(0)=1. What is C?

(c) Give an example of a pair of functions that satisfy parts (a) and (b).

56. Assume that f and g are differentiable on the interval (-c, c) and f(0)=g(0).

(a) Show that if f'(x)>g'(x) for all x∈(0, c), then f(x)>g(x) for all x∈(0, c).

(b) Show that if f'(x)>g'(x) for all x∈(-c, 0), then f(x)<g(x) for all x∈(-c, 0).

58. Show that $1-x^2/2<\cos x$ for all x∈ $(0,\infty)$.

【章節 4.3】

p.173 (17、20、28、32、35、39、42)

Exercises 1-28. Find the critical points and the local extreme values.

17. f(x) = $x^2\sqrt[3]{2+x}$.

20. f(x) = $x^{7/3}-7x^{1/3}$.

28. f(x) = $2\sin^3 x-3\sin x$, $0<x<\pi$.

32. Prove the validity of the second-derivative test in the case that f''(c)<0.

35. Find the critical points and the local extreme values of the polynomial.

P(x) = $x^4-8x^3+22x^2-24x+4$.

Show that the equation P(x) = 0 has exactly two real roots, both positive.

39. Find a and b given that f(x) = $ax/(x^2+b^2)$ has a local minimum at $x=-2$ and $f'(0)=1$.

42. Let y=f(x) be differentiable and suppose that the graph of f does not pass through the origin. The distance D from the origin to a point P(x, f(x)) of the graph is given by $D=\sqrt{x^2+[f(x)]^2}$. Show that if D has a local extreme value at c, then the line through (0, 0) and (c, f(c)) is perpendicular to the line tangent to the graph of f at (c, f(c)).

【章節 4.4】

p.180 (4、11、14、36、37、39、40)

Exercises 1-30. Find the critical points. Then find and classify all the extreme values.

4. $f(x) = 2x^2 + 5x - 1$, $x \in [-2,0]$.

11. $f(x) = \frac{x}{4+x^2}$, $x \in [-3, 1]$.

14. $f(x) = x\sqrt{4 - x^2}$.

36. Let r be a rational number, r>1, and set $f(x) = (1 + x)^r - (1 + rx)$ for x≧-1. Show that 0 is a critical point for f and show that f(0)=0 is the absolute minimum value.

37. Suppose that c is a critical point for f and f'(x)>0 for x≠c. Show that if f(c) is a local maximum, then f is not continuous at c.

39. Suppose that f is continuous on [a, b] and f(a)=f(b). Show that f has at least one critical point in (a, b).

40. Suppose that $c_1 < c_2$ and that f takes on local maxima at c_1 and c_2. Prove that if f is continuous on $[c_1, c_2]$, then there is at least one point c in (c_1, c_2) at which f takes on a local minimum.

※補充題：

I. Prove that $\lim_{x\to\infty}\left(\frac{1}{x^2}\right)$ = 0.

II. Prove that $\lim_{x\to-\infty}\left(\sqrt[3]{x}\right) = -\infty$.

【章節 4.5】

p.194(7、40、44、46)

Exercises 5-22. Describe the concavity of the graph and find the points of inflection(if any)

7. $f(x) = x^3 - 3x + 2$.

40. Find c given that the graph of $f(x) = cx^2 + x^{-2}$ has a point of inflection at (1, f(1)).

44. Find necessary and sufficient conditions on A and B for $f(x) = Ax^2 + Bx + C$

(a) to decrease between A and B with graph concave up.

(b) to increase between A and B with graph concave down.

46. Set $f(x) = \sin x$. Show that the graph of f is concave down above the x-axis and concave up below the x-axis. Does $g(x) = \cos x$ have the same property?

【章節 4.8】

p.208(4、14、21)

Exercises 1-54. Sketch the graph of the function using the approach presented in this section.

4. $f(x) = x^3 - 9x^2 + 24x - 7.$

14. $f(x) = \frac{1}{4}x - \sqrt{x},\ x \in [0,9].$

21. $f(x) = \frac{x^2}{x^2+4}.$

※補充題：Let f: (a,b) →ℝ be twice differentiable. If f’’>0 on (a,b), then the graph of y = f(x) lies above any of its tangent line.

【章節 5.2】

p.245 (12、15、17、21、23、31、39)

12. (a) Given that P = $\{x_0, x_1, \dots, x_n\}$ is an arbitrary partition of [a,b], find $L_f(P)$ and $U_f(P)$ for $f(x) = x + 3$.

(b) Use your answers to part (a) to evaluate $\int_a^b f(x)dx.$

Exercises 15-18. Express the limit as a definite integral over the indicated interval.

15.

$$\lim_{\|P\|\to 0} [(x_1^2 + 2x_1 - 3)\Delta x_1 + (x_2^2 + 2x_2 - 3)\Delta x_2 + \cdots + (x_n^2 + 2x_n - 3)\Delta x_n]; \quad [-1,2]$$

17.

$$\lim_{\|P\|\to 0} [(t_1^*)^2 \sin(2t_1^* + 1)\,\Delta t_1 + (t_2^*)^2 \sin(2t_2^* + 1)\,\Delta t_2 + \cdots + (t_n^*)^2 \sin(2t_n^* + 1)\,\Delta t_n]$$

Where $t_i^* \in [t_{i-1}, t_i]$, $i = 1,2, \dots, n$; $[0,2\pi]$

21. Let $f(x) = 2x, x \in [0,1]$. Take $P = \{0,\ \frac{1}{8},\ \frac{1}{4},\ \frac{1}{2},\ \frac{3}{4},\ 1\}$ and set $x_1^* = \frac{1}{16}, x_2^* = \frac{3}{16}, x_3^* = \frac{3}{8}, x_4^* = \frac{5}{8}, x_5^* = \frac{3}{4}$. Calculate the following:

(a)$L_f(P)$. (b)$S^*(P)$. (c)$U_f(P)$.

23. Evaluate $\int_0^1 x^3 dx$ using upper and lower sums. HINT: $b^4 - a^4 = (b^3 + b^2a +$

$ba^2 + a^3)(b - a)$.

31. A partition P=$\{x_0, x_1, x_2, \cdots, x_{n-1}, x_n\}$ of [a, b] is said to be regular if the subintervals $[x_{i-1}, x_i]$ all have the same length $\Delta x = (b - a)/n$. Let P=$\{x_0, x_1, x_2, \cdots, x_{n-1}, x_n\}$ be a regular partition of [a, b]. Show that if f is continuous and increasing on [a, b], then $U_f(P) - L_f(P) = [f(b) - f(a)]\Delta x$.

39. Let f be a function continuous on [a, b]. Show that if P is a partition of [a, b], then $L_f(P), U_f(P)$, and $\frac{1}{2}[L_f(P) + U_f(P)]$ are all Riemann sums.

【章節 5.3】

p.252 (1、3、29、31、35、36)

1. Given that $\int_0^1 f(x)dx = 6$, $\int_0^2 f(x)dx = 4$, $\int_2^5 f(x)dx = 1$,

find the following:

(a)$\int_0^5 f(x)dx$. (b)$\int_1^2 f(x)dx$. (c)$\int_1^5 f(x)dx$.

(d)$\int_0^0 f(x)dx$. (e)$\int_2^0 f(x)dx$. (f)$\int_5^1 f(x)dx$.

3. Use upper and lower sums to show that $0.5 < \int_1^2 \frac{dx}{x} < 1$.

29. Set $F(x) = 2x + \int_0^x \frac{\sin 2t}{1+t^2} dt$. Determine

(a) F(0). (b) F'(0). (c) F''(0).

31. Assume that f is continuous and $\int_0^x f(t)dt = \frac{2x}{4+x^2}$.

(a) Determine f(0).

(b) Find the zeros of f , if any.

35. Let f be continuous on [a, b]. For each x∈[a, b] set $F(x) = \int_c^x f(t)dt$, and $G(x) = \int_d^x f(t)dt$ taking c and d from [a, b].

(a) Show that F and G differ by a constant.

(b) Show that F(x)-G(x)=$\int_c^d f(t)dt$.

36. Let f be everywhere continuous and set $\mathrm{F(x)} = \int_0^x [t \int_1^t f(u)du]dt$. Find

(a) F'(x). (b) F'(1). (c)F''(x). (d)F''(1).

p.284 (16、24)

16. Derive a formula for $\frac{d}{dx}(\int_{u(x)}^{b} f(t)dt)$ given that u is differentiable and *f* is continuous.

24. Show that $\frac{d}{dx}\left(\int_{u(x)}^{v(x)} f(t)dt\right) = f\big(v(x)\big)v'(x) - f(u(x))u'(x)$ given that u and v are differentiable and *f* is continuous.

※補充題：Let $\mathrm{f(x)} = \int_{x^2+2x}^{\sec x^2}(t^3 - \frac{1}{t} + 2)dt$. Find f'(x).

【章節 5.4】

p.258 (8、14、16、26、32、49、61、62、63、64)

8. Evaluate the integral $\int_1^2 \left(\frac{3}{x^3} + 5x\right) dx$.

14. Evaluate the integral $\int_0^1 (x^{3/4} - 2x^{1/2})dx$.

16. Evaluate the integral $\int_0^a (a^2x - x^3)dx$.

26. Evaluate the integral $\int_{\pi/6}^{\pi/3} \sec x \tan x \, dx$.

32. Evaluate the integral $\int_{\pi/4}^{\pi/2} \csc x \, (\cot x - 3 \csc x)dx$.

49. Determine whether the calculation is valid. If it is not valid, explain why it is not valid. $\int_0^{2\pi} x \cos x \, dx = [x \sin x + \cos x]_0^{2\pi} = 1 - 1 = 0$.

61. *(Important)* If f is a function and its derivative f' is continuous on [a, b], then $\int_a^b f'(t)dt = f(b) - f(a)$. Explain the reasoning here.

62. Let f be a function such that f' is continuous on [a, b]. Show that $\int_a^b f(t)f'(t)dt = \frac{1}{2}[f^2(b) - f^2(a)]$.

63. Given that f has a continuous derivative, compare $\frac{d}{dx}[\int_a^x f(t)dt]$ to $\int_a^x \frac{d}{dt}[f(t)]dt$.

64. Given that f is a continuous function, set $\mathrm{F(x)} = \int_0^x xf(t)dt$. Find F'(x). HINT: The answer is not xf(x).

p.289 (15、20、21、22)

15. Suppose that *f* has a continuous derivative on [a, b]. What is the average value of f' on [a, b]?

20. Let *f* be continuous. Show that, if *f* is an odd function, then its average value on every interval of the form [-a, a] is zero.

21. Suppose that f is continuous on [a, b] and $\int_a^b f(x)dx = 0$. Prove that there is at least one number c in (a, b) for which f(c)=0.

22. Show that the average value of the functions $\mathrm{f(x)} = \sin \pi x$ and $\mathrm{g(x)} = \cos \pi x$ is zero on every interval of length 2n, n a positive integer.

【章節 5.5】

p.265 (5、22、27、29、35)

5. Find the area between the graph of f and the x-axis. f(x)=$(2x^2+1)^2$, x∈[0, 1].

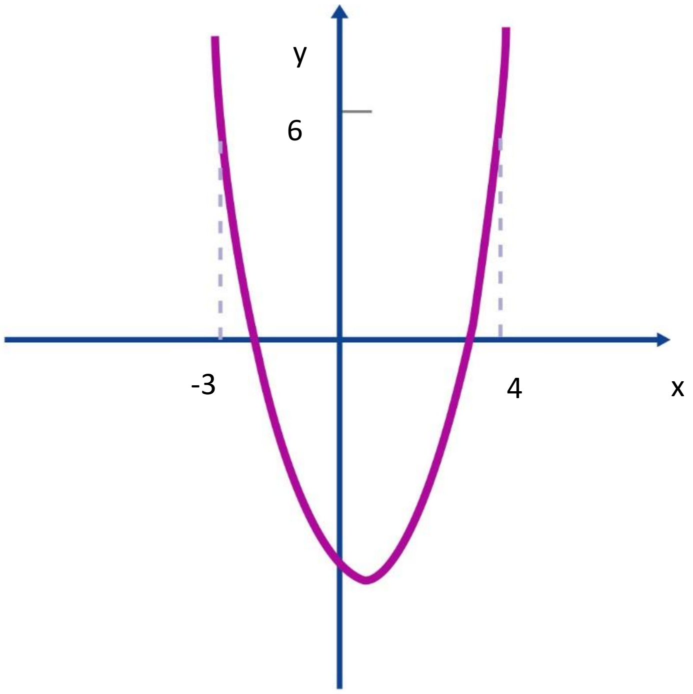
22. Sketch the region bounded by the curves and find its area. $y = x^2$, y=$-\sqrt{x}$, x=4.

27. The graph of $f(x) = x^2 - x - 6$ is shown in the accompanying figure.

(a) Evaluate $\int_{-3}^{4} f(x)dx$ and interpret the result in terms of areas,

(b) Find the area between the graph of f and the x-axis from x=-3 to x=4,

(c) Find the area between the graph of f and the x-axis from x=-2 to x=3.



29. Set f(x)=$x^3 - x$.

(a) Evaluate $\int_{-2}^{2} f(x)dx$.

(b) Sketch the graph of f and find the area between the graph and the x-axis from x=-2 to x=2.

35. Sketch the region bounded by the x-axis and the curves $y = \sin x$ and $y = \cos x$ with x∈ $[0, \pi/2]$, and find its area.

【章節 5.6】

p.273 (10、26、33)

10. Calculate $\int (t^2-a)(t^2-b)dt.$

26. Find f from the information given. $\mathrm{f}''(\mathrm{x}) = -12\mathrm{x}^2, f(0)=1, f(0)=2.$

33. Compare $\frac{d}{dx}[\int f(x)dx]$ to $\int \frac{d}{dx}[f(x)]dx.$

※補充題：

①Show that $\int \frac{\sin x^2 - 2x^2\cos x^2}{sin^2 x^2}dx = \frac{x}{\sin x^2} + c.$

②$\int [f(x)g''(x) - f''(x)g(x)]dx = f(x)g'(x) - f'(x)g(x) + c$

【章節 5.7】

p.279 (1、2、10、22、26、31、47、49、56、58、63、68、73、84)

1. Calculate $\int \frac{dx}{(2-3x)^2}.$

2. Calculate $\int \frac{dx}{\sqrt{2x+1}}.$

10. Calculate $\int x^{n-1}\sqrt{a+bx^n}dx.$

22. Evaluate $\int_{-1}^{0} 3x^2(4+2x^3)^2dx.$

26. Evaluate $\int_{-a}^{0} y^2(1-\frac{y^3}{a^2})^{-2}dy.$

31. Calculate $\int x\sqrt{x+1}dx.$ [set u=x+1]

47. Calculate $\int cos^4 x \sin x\, dx.$

49. Calculate $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx.$

56. Calculate $\int (1+tan^2 x)sec^2 xdx.$

58. Calculate $\int xsin^4(x^2-\pi)\cos(x^2-\pi)\,dx.$

63. Calculate $\int x^2\tan(x^3+\pi)sec^2(x^3+\pi)\,dx.$

68. Evaluate $\int_0^1 cos^2\frac{\pi}{2}x \sin\frac{\pi}{2}x\, dx.$

73. Calculate $\int cos^2 5xdx.$

84. Let f be a continuous function, c a real number. Show that

(a) $\int_{a+c}^{b+c} f(x-c)dx = \int_a^b f(x)dx,$

and, if c≠0,

(b) $\frac{1}{c}\int_{ac}^{bc} f(x/c)dx = \int_a^b f(x)dx.$

【章節 5.8】

p.284 (30、33、34)

30. *(Important)* Prove that, if f is continuous on [a, b] and $\int_a^b |f(x)|dx = 0$, then

f(x)=0 for all x in [a, b]. HINT: Exercise 50, Section 2.4.

33. (a) Let f be continuous on [-a, 0]. Use a change of variable to show that

$\int_{-a}^{0} f(x)dx = \int_0^a f(-x)dx.$

(b) Let f be continuous on [-a, a]. Show that $\int_{-a}^{a} f(x)dx = \int_0^a [f(x) + f(-x)]dx.$

34. Let f be a function continuous on [-a, a]. Prove the statement basing your argument on Exercise 33.

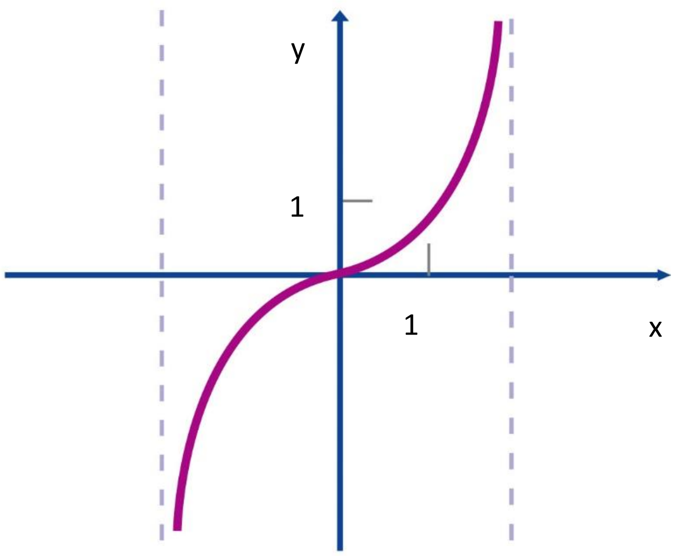
(a) $\int_{-a}^{a} f(x)dx = 0$ if f is odd.

(b) $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if f is even.

【章節 7.1】

p.340 (31、32、33、34、42、46、49、52、53)

31. Sketch the graph of the inverse of the function graphed below.



32. (a) Show that the composition of two one-to-one functions, f and g , is one-to-one.

(b) Express $(f \circ g)^{-1}$ in terms of f^{-1} and g^{-1}.

33. (a) Let f(x)=$\frac{1}{3}x^3 + x^2 + kx$, k a constant. For what values of k is f one-to-one?

(b) Let g(x)=$x^3 + kx^2 + x$, k a constant. For what values of k is g one-to-one?

34. (a) Suppose that f has an inverse, f(2)=5, and f'(2)=$-\frac{3}{4}$. What is $(f^{-1})'(5)$?

(b) Suppose that f has an inverse, f(2)=-3, and f'(2)=$\frac{2}{3}$. If g=$1/f^{-1}$, what is g'(-3)?

42. Verify that f has an inverse and find $(f^{-1})'(c)$. $f(x) = x^5 + 2x^3 + 2x;$ c=-5.

46. Find a formula for $(f^{-1})'(x)$ given that f is one-to-one and its derivative satisfies the equation given. $f'(x) = 1 + [f(x)]^2$.

49. Let $f(x) = \frac{ax+b}{cx+d}$.

(a) Show that f is one-to-one iff ad-bc≠0.

(b) Suppose that ad-bc≠0. Find f^{-1}.

52. Set $f(x) = \int_1^{2x} \sqrt{16 + t^4}\,dt$.

(a) Show that f has an inverse.

(b) Find $(f^{-1})'(0)$.

53. Let f be a twice differentiable one-to-one function and set g=f^{-1}.

(a) Show that $g''(x) = -\frac{f''[g(x)]}{(f'[g(x)])^3}$.

(b) Suppose that the graph of f is concave up (down). What can you say then about the graph of f^{-1}?

【章節 7.2】

p.346 (22、23、24、25)

22. Solve the equation for x. 2ln(x+2)$-\frac{1}{2}$lnx^4=1.

23. Show that $\lim_{x\to 1}\frac{\ln x}{x-1}=1$. HINT: Note that $\frac{\ln x}{x-1}=\frac{\ln x-\ln 1}{x-1}$ and interpret the limit as a derivative.

24. Let n be a positive integer greater than 2. Draw relevant figures. Find the greatest integer k for which $\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{k}<\ln n$.

25. Let n be a positive integer greater than 2. Draw relevant figures. Find the least integer k for which $\ln n<1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{k}$.

【章節 7.3】

p.354 (8、12、14、22、23、24、27、31、32、33、34)

8. Determine the domain and find the derivative. $\mathrm{f(x)}=\ln(\ln x)$.

12. Determine the domain and find the derivative. $\mathrm{f(x)}=\ln\sqrt[4]{x^2+1}$.

14. Determine the domain and find the derivative. $\mathrm{f(x)}=\cos(\ln x)$.

22. Calculate $\int\frac{csc^2x}{2+\cot x}dx$.

23. Calculate $\int\frac{x}{(3-x^2)^2}dx$.

24. Calculate $\int\frac{\ln(x+a)}{x+a}dx$.

27. Calculate $\int\frac{1}{x\ln x}dx$.

31. Calculate $\int\frac{\sin x-\cos x}{\sin x+\cos x}dx$.

32. Calculate $\int\frac{1}{\sqrt{x}(1+\sqrt{x})}dx$. HINT: Set u=1+$\sqrt{x}$.

33. Calculate $\int\frac{\sqrt{x}}{1+x\sqrt{x}}dx$.

34. Calculate $\int\frac{\tan(\ln x)}{x}dx$.

【章節 7.4】

p.362 (18、24、31、40、41、42、47、49、72)

18. Differentiate y=$\frac{e^{2x}-1}{e^{2x}+1}$.

24. Differentiate $f(x) = \ln(\cos e^{2x})$

31. Calculate $\int \frac{e^{1/x}}{x^2}dx.$

40. Calculate $\int \frac{\sin(e^{-2x})}{e^{2x}}dx.$

41. Calculate $\int \cos x e^{\sin x}\, dx.$

42. Calculate $\int e^{-x}[1+\cos(e^{-x})]dx.$

47. Evaluate $\int_0^1 \frac{e^x+1}{e^x}dx.$

49. Evaluate $\int_0^{\ln 2} \frac{e^x}{e^x+1}dx.$

72. Prove that for all x>0 and all positive integers n

$$e^x > 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}.$$

Recall that $n! = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1.$

HINT: $e^x = 1 + \int_0^x e^t dt > 1 + \int_0^x dt = 1 + x$

$e^x = 1 + \int_0^x e^t dt > 1 + \int_0^x (1+t)dt = 1 + x + \frac{x^2}{2}$, and so on.

【章節 7.7】

p.385 (6、8、18、24、25、35、36、45、47、48、49、50、53、57、61)

6. Determine the exact value. (a)$\sin^{-1}(\sin[11\pi/6])$; (b)$\tan^{-1}(\tan[11\pi/4])$.

8. Determine the exact value. (a)$\cos(\sin^{-1}[\frac{3}{5}])$; (b)$\sec(\tan^{-1}[\frac{4}{3}])$.

18. Differentiate $v = \tan^{-1} e^x$.

24. Differentiate g(x)=$\sec^{-1}(\cos x + 2)$.

25. Differentiate $\theta = \sin^{-1}(\sqrt{1-r^2})$.

35. Calculate $\int \frac{1}{\sqrt{a^2-(x+b)^2}}dx$ taking a>0. HINT: Set au=x+b.

36. Calculate $\int \frac{1}{a^2+(x+b)^2}dx$ taking a>0.

45. Evaluate $\int_0^{3/2} \frac{dx}{9+4x^2}$.

47. Evaluate $\int_{3/2}^{3} \frac{dx}{x\sqrt{16x^2-9}}$.

48. Evaluate $\int_4^6 \frac{dx}{(x-3)\sqrt{x^2-6x+8}}$.

49. Evaluate $\int_{-3}^{-2} \frac{dx}{\sqrt{4-(x+3)^2}}$.

50. Evaluate $\int_{\ln 2}^{\ln 3} \frac{e^x}{\sqrt{1-e^{-2x}}} dx$.

53. Calculate $\int \frac{x}{\sqrt{1-x^4}} dx$.

57. Calculate $\int \frac{sec^2 x}{9+tan^2 x} dx$.

61. Calculate $\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$.

【章節 8.1】

※補充題：① $\int \frac{dx}{\sqrt{3-4x^2}} =?$ ② $\int \frac{dx}{\sqrt{e^{2x}-6}} =?$ ③ $\int \frac{dx}{\sqrt{4x-x^2}} =?$ ④ $\int \frac{dx}{4x^2+4x+2} =?$

【章節 8.2】

p.408 (4、7、21、29、32、33、34、37、42、44)

4. Calculate $\int x \ln x^2 \, dx$.

7. Calculate $\int \frac{x^2}{\sqrt{1-x}} dx$.

21. Calculate $\int x^2(x+1)^9 dx$.

29. Calculate $\int x^3 \sin x^2 \, dx$.

32. Calculate $\int \frac{\sin^{-1} 2x}{\sqrt{1-4x^2}} dx$.

33. Calculate $\int_0^1 x \tan^{-1} x^2 \, dx$.

38. Calculate $\int \cos(\ln x) \, dx$.   HINT: Integrate by parts twice.

40. Calculate $\int_1^{2e} x^2 (\ln x)^2 dx$.

45. Derive the following formula $\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2} + C$.

68. Let n be a positive integer. Show that $\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1} dx$.

The formula given in Exercise 67 reduces the calculation of $\int x^n e^{ax} dx$ to the calculation of $\int x^{n-1} e^{ax} dx$. The formula given in Exercise 68 reduces the calculation of $\int (\ln x)^n dx$ to the calculation of $\int (\ln x)^{n-1} dx$. Formulas (such as these) which reduce the calculation of an expression in n to the calculation of the corresponding expression in n-1 are called reduction formulas.

74. If P is a polynomial of degree k, then $\int P(x)e^x dx = [P(x) - P'(x) + \cdots \pm P^{(k)}(x)]e^x + C$. Verify this statement. For simplicity, take k=4.

76. Use integration by parts to show that if f has an inverse with continuous first derivative, then $\int f^{-1}(x)dx = xf^{-1}(x) - \int x(f^{-1})'(x)dx$.

77. Show that if f and g have continuous second derivatives and f(a)=g(a)=f(b)=g(b)=0, then $\int_a^b f(x)g''(x)dx = \int_a^b g(x)f''(x)dx$.

78. You are familiar with the identity $\mathrm{f(b)} - \mathrm{f(a)} = \int_a^b f'(x)dx$.

(a) Assume that f has a continuous second derivative. Use integration by parts to derive the identity $\mathrm{f(b)} - \mathrm{f(a)} = \mathrm{f'(a)(b-a)} - \int_a^b f''(x)(x-b)dx$.

(b) Assume that f has a continuous third derivative. Use the result in part (a) and integration by parts to derive the identity $\mathrm{f(b)} - \mathrm{f(a)} = \mathrm{f'(a)(b-a)} + \frac{f''(a)}{2}\mathrm{(b-a)^2} - \int_a^b \frac{f'''(x)}{2}(x-b)^2 dx$.

Going on in this manner, we are led to what are called Taylor series (Chapter 12).

【章節 8.3】

p.415 (2、8、16、27、29、31、33、34、37、42、44)

2. Calculate $\int_0^{\pi/8} cos^2 4xdx$. (If you run out of ideas, use the examples as models.)

8. Calculate $\int sin^2 xcos^4 xdx$. (If you run out of ideas, use the examples as models.)

16. Calculate $\int_0^{\pi/2} \cos 2x \sin 3x\, dx$. (If you run out of ideas, use the examples as models.)

27. Calculate $\int \sin 5x \sin 2x\, dx$. (If you run out of ideas, use the examples as models.)

29. Calculate $\int sin^{5/2} x\, cos^3 xdx$. (If you run out of ideas, use the examples as models.)

31. Calculate $\int tan^5 3xdx$. (If you run out of ideas, use the examples as models.)

33. Calculate $\int_{-1/6}^{1/3} sin^4 3\pi x cos^3 3\pi xdx$. (If you run out of ideas, use the examples as models.)

34. Calculate $\int_0^{1/2} \cos \pi x \cos \frac{1}{2}\pi x\, dx$. (If you run out of ideas, use the examples as models.)

37. Calculate $\int tan^4 xsec^4 xdx$. (If you run out of ideas, use the examples as models.)

42. Calculate $\int_{\pi/4}^{\pi/2} csc^3 x \cot x\, dx$ (If you run out of ideas, use the examples as models.)

44. Calculate $\int_0^{\pi/3} \tan x\, sec^{3/2} xdx$ (If you run out of ideas, use the examples as models.)

【章節 8.4】

p.421 (13、14、21、26、31、32)

13. Calculate $\int_0^5 x^2\sqrt{25-x^2}dx$.

14. Calculate $\int \frac{\sqrt{1-x^2}}{x^4}dx$.

21. Calculate $\int \frac{dx}{x^2\sqrt{a^2+x^2}}$.

26. Calculate $\int \frac{e^x}{\sqrt{9-e^{2x}}}dx$.

31. Calculate $\int x\sqrt{6x-x^2-8}dx$.

32. Calculate $\int \frac{x+2}{\sqrt{x^2+4x+1}}dx$.

【章節 8.5】

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11. Calculate $\int \frac{2x^4-4x^3+4x^2+3}{x^3-x^2}dx$.

20. Calculate $\int \frac{2x-1}{(x+1)^2(x-2)^2}dx$.

23. Calculate $\int \frac{x^3+4x^2-4x-1}{(x^2+1)^2}dx$.

28. Calculate $\int \frac{1}{(x-1)(x^2+1)^2}dx$.

34. Evaluate $\int_0^2 \frac{x^3}{(x^2+2)^2}dx$.