微積分（D）附件

【章節 11 Sequences；Indeterminate forms】

【part 1】

1. Find the limit.

   ① $\lim_{\theta\to\frac{\pi}{2}}\frac{1-\sin\theta}{\csc\theta}$ ② $\lim_{x\to 0}\frac{x}{\tan^{-1}(4x)}$ ③ $\lim_{x\to\infty}\left(x^3e^{-x^2}\right)$

   ④ $\lim_{x\to 0^+}(\sin x\ln x)$ ⑤ $\lim_{x\to\infty}(x\tan\frac{1}{x})$ ⑥ $\lim_{x\to\infty}(xe^x-x)$

2. If f' is cont., f(2)=0 and $f'(2)$=7, evaluate $\lim_{x\to 0}\frac{f(2+3x)+f(2+5x)}{x}$.

3. If f' is cont., use $\mathrm{l'Hospital}$ Rule to show that $\lim_{h\to 0}\frac{f(x+h)-f(x-h)}{2h}=f'(x)$.

【part 2】

1. Determine whether the sequence converges or diverges. If it converges, find the limit.

   ① $a_n=\frac{3+5n^2}{n+n^2}$ ② $a_n=\frac{2^n}{3^{n+1}}$ ③ $a_n=\frac{(n+2)!}{n!}$ ④ $a_n=\frac{(-1)^n n^3}{n^3+2n^2+1}$

   ⑤ $a_n=\cos(\frac{2}{n})$ ⑥ $a_n=\frac{\cos n}{2^n}$ ⑦ $\{n\cos(n\pi)\}$ ⑧ {0,1,0,0,1,0,0,0,1⋯}

   ⑨ $a_n=\frac{(\ln n)^2}{n}$

2. ⓐ If $\{a_n\}$ is convergent, show that $\lim_{n\to\infty}a_{n+1}=\lim_{n\to\infty}a_n$

   ⓑ A sequence $\{a_n\}$ is defined by $a_1=1$ and $a_{n+1}=\frac{1}{1+a_n}$ $\forall n\geqq 1$.

   Assuming that $\{a_n\}$ is convergent, find its limit.

3. A sequence $\{a_n\}$ is given by $a_1=\sqrt{2}$, $a_{n+1}=\sqrt{2+a_n}$.

   ⓐ By induction, show that $\{a_n\}$ is increasing and bounded above by 3.

   ⓑ Show that $\lim_{n\to\infty}a_n$ exists and find its limit.

4. ⓐ Show that the sequence defined by $a_1=2$, $a_{n+1}=\frac{1}{3-a_n}$

   satisfies $0\leqq a_n\leqq 2$ and is decreasing.

   ⓑ Show that it converges and find its limit.