Quiz 4

(a) A thick slab extending from $z=-a$ to $z=+a$ (and infinite in the x and y directions) carries a uniform volume current $\mathbf{J}=J\hat{\mathbf{x}}$. (Prob. 5.15)

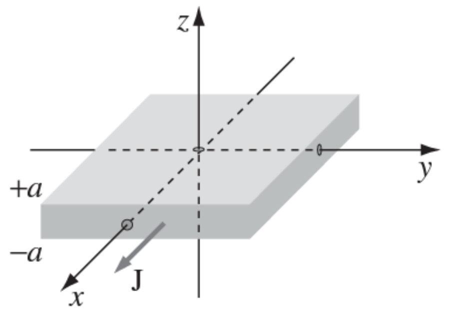
(1) What are the directions of the magnetic field inside and outside the slab? Explain.

(2) Find the magnetic field as a function of z, both inside and outside the slab.

For a infinite sheet of surface current: $\mathbf{B}=\dfrac{\mu_0\mathbf{K}\times\hat{\mathbf{n}}}{2}$ (By right-hand rule and symmetry)

Dividing this slab into sheets, $\mathbf{K}\to\mathbf{J}$, knowing that:

$$\mathbf{B}(z)=\int\frac{\mu_0\mathbf{J}\times\hat{\mathbf{n}}}{2}dz'=\begin{cases}\dfrac{\mu_0J(2a)}{2}(-\hat{\mathbf{y}})=\mu_0Ja(-\hat{\mathbf{y}}) & z>a\\ \dfrac{\mu_0J(a-z)}{2}(+\hat{\mathbf{y}})+\dfrac{\mu_0J(z+a)}{2}(-\hat{\mathbf{y}})=\mu_0Jz(-\hat{\mathbf{y}}) & a>z>-a\\ \mu_0Ja(+\hat{\mathbf{y}}) & -a>z\end{cases}$$

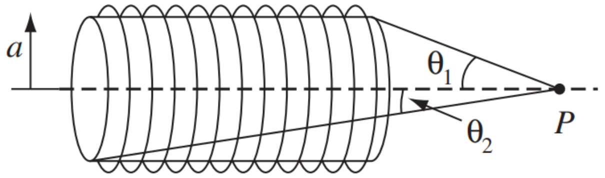


Note that $\mathbf{B}(z=0)=0$

(b) A solenoid of length L and radius a has N turns of wire and carries a current I.

(1) Find the magnetic flux density (magnetic field strength) at a point P along the central axis. (Prob. 5.11)

(2) Suppose the length of the solenoid is now infinitely long. Find the magnetic vector potential **A** both inside and outside the solenoid. Also, verify that the answers satisfy the boundary condition of **A**. (Ex 5.12)



Eq(5.41): $\mathbf{B}(z)=\dfrac{\mu_0I}{2}\dfrac{R^2}{\left(R^2+z^2\right)^{3/2}}\hat{\mathbf{z}}$ $\qquad n=\dfrac{N}{L}$

$$\mathbf{B}(z)=n\times\frac{\mu_0I}{2}\int_{a\cot\theta1}^{a\cot\theta_2}\frac{a^2dz}{\left(a^2+z^2\right)^{3/2}}\hat{\mathbf{z}}=\hat{\mathbf{z}}\frac{\mu_0nI}{2}\int_{\theta1}^{\theta_2}\frac{a^2\left(\dfrac{-a}{\sin^2\theta}d\theta\right)}{\left(a^2+a^2\cot^2\theta\right)^{3/2}}$$

$$=\hat{\mathbf{z}}\frac{\mu_0nI}{2}\int_{\theta1}^{\theta_2}\frac{a^2\left(\dfrac{-a}{\sin^2\theta}d\theta\right)}{\left(\dfrac{a^2}{\sin^2\theta}\right)^{3/2}}=\hat{\mathbf{z}}\frac{\mu_0nI}{2}\int_{\theta1}^{\theta_2}-\sin\theta d\theta=\frac{\mu_0nI}{2}\left(\cos\theta_2-\cos\theta_1\right)\hat{\mathbf{z}}$$

$=\mu_0nI\hat{\mathbf{z}}$ for an infinite solenoid, $\theta_2=0,\ \theta_1=\pi$, so $\cos\theta_2-\cos\theta_1=2$

$$\oint\mathbf{A}\cdot d\mathbf{l}=A(2\pi s)=\int\mathbf{B}\cdot d\mathbf{a}=\begin{cases}\mu_0nI\left(\pi s^2\right) & s\le R\\ \mu_0nI\left(\pi R^2\right) & s\ge R\end{cases}$$

$$\Rightarrow\mathbf{A}=\begin{cases}\dfrac{\mu_0nIs}{2}\hat{\phi} & s\le R\\ \dfrac{\mu_0nIR^2}{2s}\hat{\phi} & s\ge R\end{cases}$$

B.C.1 $\quad\mathbf{A}_{s\le R}(s=R)=\dfrac{\mu_0nIR}{2}\hat{\phi}=\mathbf{A}_{s\ge R}(s=R)$

B.C.2 $\quad\left.\dfrac{\partial\mathbf{A}_{s\le R}}{\partial s}\right|_{s=R}-\left.\dfrac{\partial\mathbf{A}_{s\ge R}}{\partial s}\right|_{s=R}=-\dfrac{\mu_0nI}{2}\hat{\phi}-\dfrac{\mu_0nI}{2}\hat{\phi}=-\mu_0nI\hat{\phi}=-\mu_0\mathbf{K}$