Midterm for Thermal Physics (II)
Date: May 7, 2012

(1) Please do not flip the sheet until instructed.
(2) Please try to be as neat as possible so that I can understand your answers without ambiguity.
(3) While it is certainly your rights to make wild guesses or memorize irrelevant details, I would truly appreciate if you try to make your answers logical.
(4) Good luck for all hard-working students!

Lecturer: Hsiu-Hau Lin

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1. Binary mixture (20%) Consider a simple model for a binary mixture $A_{1-x}B_x$ in two dimensions. The average number of surrounding neighbors is 6 and the potential energies for A−A, B−B and A−B bonds are $u_{AA} = 3\Delta$, $u_{BB} = -2\Delta$ and $u_{AB} = \Delta$ accordingly, where Δ is some positive constant. **(a)** Find the free energy $f(x)$ of the binary mixture. **(b)** Sketch the phase diagram and highlight the solubility gap.

2. Solidification of a binary alloy (20%) Consider a binary alloy $A_{1-x}B_x$ with the solidification temperatures $\tau_A > \tau_B$. For simplicity, assume neither the solid nor the liquid has a solubility gap. **(a)** Sketch the free energies $f_S(x)$ and $f_L(x)$ for the solid and the liquid at three different temperature regimes: (1) $\tau > \tau_A$ (2) $\tau_B < \tau < \tau_A$ (3) $\tau < \tau_B$. **(b)** Construct the phase diagram and explain how the solidification of a binary alloy proceeds upon cooling.

3. Minimum conductivity in semiconductor (20%) The electrical conductivity in a semiconductor is

$$\sigma = en_e\tilde{\mu}_e + en_h\tilde{\mu}_h$$

where $\tilde{\mu}_e$ and $\tilde{\mu}_h$ are the electron and hole mobilities. The quantum concentrations for conduction and valence bands are n_c and n_v with a band gap ϵ_g and the electron gas is non-degenerate. **(a)** Find the conductivity σ_{int} for an intrinsic semiconductor. **(b)** For most semiconductors, $\tilde{\mu}_e > \tilde{\mu}_h$. The minimum conductivity can be reached in a p-type semiconductor. Find the minimum conductivity σ_{min} and compare with the intrinsic conductivity σ_{int}.

4. Potential profile in $p - n$ junction (20%) Near the interface of a p−n junction, electrons and holes annihilate each other, creating a depletion zone. The width of the depletion zone of the p−type side is w_p and that on the n−type side is w_n. The charge distribution in a $p - n$ junction can be approximated as,

$$\rho(x) = \begin{cases} -en_a, & 0 < x < w_p; \\ en_d, & -w_n < x < 0; \\ 0, & \text{otherwise.} \end{cases}$$

The widths w_p, w_n need to be solved from the Poisson equation for the electrostatic potential $\varphi(x)$. The boundary conditions are $\varphi(-\infty) = 0$ and $\varphi(+\infty) = -V_{bi}$. Find the electric field $E(0)$ at the interface of the junction.

5. Joule-Thomson effect (20%) Investigate the Joule-Thomson effect in a van der Waals gas described by

$$P = \frac{N\tau}{V - Nb} - \frac{N^2a}{V^2} \approx \frac{N\tau}{V} + \left(\frac{N^2b\tau}{V^2} - \frac{N^2a}{V^2}\right),$$

where the corrections arisen from the finite volume of molecules and the inter-molecular attraction. Explain the constancy of enthalpy, ideal gas expansion and the Joule-Thomson effect for a van der Waals gas in detail.

6. Recombination of electrons and holes (Bonus 20%) Consider a semiconducting device at nanoscale. There are N_c conduction orbitals at energy ϵ_c and N_v valence orbitals at energy ϵ_v. The average electron number in the conduction orbitals is N_e and the average hole number in the valence orbitals is N_h. The decay rate for an electron tunneling from an occupied conduction orbital to an empty valence orbital is γ and the rate for the reverse process is γ'. **(a)** Compute the recombination rate $R_{c\to v}$ from the conduction orbitals to the valence orbitals and the rate $R_{v\to c}$ for the reverse processes. **(b)** Make use of detail balance in thermal equilibrium to express γ' in terms of γ. Note that Fermi-Dirac distribution should be used here to account for quantum statistics. Find the total recombination rate R for electrons and holes.