國　立　清　華　大　學　命　題　紙

95 學年度 電機領域聯合招生 系（所） _____ 組碩士班入學考試

科目 工程數學 B 科目代碼 9903 共 2 頁第 1 頁 *請在【答案卷卡】內作答

1. (12 %) Please state "TRUE" or "FALSE" for the following statements.
If you only answer "True" or "False" without complete explanation, you get 0 point. You need to explain it briefly to get full credits.
   a) If A and B are invertible matrices in $\mathbf{M}_{nxn}(F)$ and B is similar to A, then, for any integer $k > 0$, A^k and B^k are similar.
   b) Let $\mathbf{T}$: $\mathbf{R}^n \rightarrow \mathbf{R}^n$ be linear transformation. If $\mathbf{T}(x_1) = \mathbf{T}(x_2)$, then $x_1 = x_2$ when nullity($\mathbf{T}$) = 0.
   c) If a vector space $\mathbf{V}$ is the direct sum of $\mathbf{W}_1$ and $\mathbf{W}_2$, then $\mathbf{W}_1 \cap \mathbf{W}_2 = \varnothing$.
   d) $\{0\}$ is a linearly independent set.
   e) $\{1, x, x^2\}$ is an orthonormal basis for $\mathbf{P}_3(F)$.
   f) The vectors in an eigenspace of a linear operator $\mathbf{T}$ are eigenvectors of $\mathbf{T}$.

2. (13 %) If A is an n x n matrix,
   (a) Please find the required multiplications by cofactor expansion along the 1st row to calculate its determinant. (3 %)
   (b) How many multiplications do we need to calculate if we apply the elementary row operations in calculating the determinant? (6 %)
   (c) From (a) and (b), if you need to write a numerical program to calculate a matrix's determinant, which method do you prefer? Please justify your answer. (4 %)

3. (10%) Let A be an $n \times n$ matrix that is similar to a lower triangular matrix and has the distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_k$ with corresponding multiplicities $m_1, m_2, \ldots, m_k$. What are tr(A) and det(A)?

4. (15%) In R^4, let $S = \{u_1, u_2, u_3\}$, where $u_1 = (1, 0, 1, 0)$, $u_2 = (1, 1, 1, 1)$, and $u_3 = (0, 1, 2, 1)$. Use the Gram-Schmidt process and compute an orthonormal basis $\{v_1, v_2, v_3\}$ for the subspace span(S).

5. (20%) A company puts six types of collectable into their product boxes, one in each box and in equal proportions. If a customer decides to collect all six of the collectable, what is the expected number of the product boxes that he or she should buy?

6. (8%) How many different ways can you put 5 identical beads into 5 different boxes?

7. (7%) In a study it was discovered that 30% of the paintings of a certain gallery are not original. A collector in 15% of the cases makes a mistake in judging if a painting is authentic or a copy. If she buys a piece thinking that it is original, what is the probability that it is not?

8 (15%) Suppose the counts recorded by a Geiger counter follow a Poisson process with an average of 3 counts per minute.

(a) What is the probability that there are no counts in a 20 seconds interval?

(b) What is the probability that the first count occurs in less than 10 seconds?

(c) Suppose there is no counts in the first minute, what is the probability that first count occurs in the next minute?

Poisson distribution :

Pr*obability mass function*

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0, 1, 2, 3, \ldots.$$

- *mean* $E(X) = \lambda$.

where X = No. of counts in a time interval.