科目：工程數學 B(5004)

校系所組：中央大學通訊工程學系（甲組）
中央大學電機工程學系(電子組、系統與生醫組)
交通大學電子研究所(甲組、乙組)
交通大學電控工程研究所(甲組、丙組)
交通大學電信工程研究所（甲組）
清華大學電機工程學系(乙組、丙組、丁組)
清華大學通訊工程研究所

1. (10 %) Consider the following linear equation system in R:

$$\begin{cases} x+ay+3z=2 \\ x+2y+2z=3 \\ x+3y+az=a+3 \end{cases}$$

Please discuss and determine all possible values of a and find its corresponding solution set conditions.

2. Let α, β, and γ be the ordered basis of $\mathbf{P}_2(R)$, $\mathbf{P}_3(R)$, and $\mathbf{M}_{2\times 2}(R)$, respectively defined by

$$\alpha=\left\{1, x, \frac{1}{2}(-1+3x^2)\right\}, \quad \beta=\left\{1, x, \frac{1}{2}(-1+3x^2), \frac{1}{2}(-3x+5x^3)\right\}$$

and

$$\gamma=\left\{\begin{pmatrix}1&0\\0&0\end{pmatrix}, \begin{pmatrix}2&1\\0&0\end{pmatrix}, \begin{pmatrix}3&2\\1&0\end{pmatrix}, \begin{pmatrix}4&3\\2&1\end{pmatrix}\right\}$$

Let $\mathbf{T}: \mathbf{P}_2(R) \to \mathbf{P}_3(R)$, and $\mathbf{U}: \mathbf{P}_3(R) \to \mathbf{M}_{2\times 2}(R)$ be the linear transformations respectively defined by

$$\mathbf{T}(f(x))=15xf(x) \quad \text{and} \quad \mathbf{U}(g(x))=2\begin{pmatrix}g(4)&g(3)\\g(2)&g(1)\end{pmatrix}$$

(a) (5%) Compute the matrix representation of $\mathbf{T}$ in α and β.

(b) (5%) Compute the matrix representation of $\mathbf{U}$ in β and γ.

(c) (5%) Find a basis for the null space of $\mathbf{U}$.

(d) (5%) Compute the nullity and rank of $\mathbf{UT}$.

3. Let $\mathbf{T}$ be a linear operator on the inner product space R^4 defined by $\mathbf{T}(\upsilon)=A\upsilon$, for all $\upsilon \in \mathrm{R}^4$, where

$$A=\begin{pmatrix}4&5&3&0\\0&-2&-4&0\\0&5&7&0\\2&5&7&2\end{pmatrix}$$

(a) (8%) Find the eigenvalues and the corresponding eigenspaces of $\mathbf{T}$.

(b) (4%) Determine whether or not there exists an orthonormal basis of R^4 that consists of eigenvectors of $\mathbf{T}$? Explain why.

(c) (8%) Find an orthonormal basis for the $\mathbf{T}$-cyclic subspace of R^4 generated by $\upsilon=\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$

注意：背面有試題

台灣聯合大學系統 99 學年度碩士班考試命題紙

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4. (11%) Let the joint probability density function for random variables X and Y be

$$f_{XY}(x,y)=\begin{cases} c\cdot xy^2, & 0<x<y<1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) (5%) Find the constant c.

(b) (6%) Find the probability $\mathbf{P}(Y>1/2 \mid X=1/4)$.

5. (6%) Suppose that several relevant probability measures of the current quick test for H1N1 virus are known as follows: if a person is infected by H1N1, the test result is positive with probability 0.8. On the other hand, if the person doesn't receive H1N1, the test result is negative with probability 0.9. However, given a person just tested positive, the probability that the person is currently infected by H1N1 is only 0.6. What is the probability that a person randomly drawn from the population is currently infected by H1N1?

6. (8%) Let X and Y be independent exponential random variables with density function $f_X(x)=\lambda_1 e^{-\lambda_1 x}$ and $f_Y(y)=\lambda_2 e^{-\lambda_2 y}$, respectively. Let β be a Bernoulli random variable, independent of X and Y, with $\mathbf{P}(\beta=0)=\mathbf{P}(\beta=1)=1/2$. Define

$$Z=\begin{cases} \max(X,Y), & \beta=1, \\ \min(X,Y), & \beta=0. \end{cases}$$

Find the probability density function of the random variable Z.

7. (10%) The attached table provides the numerical values of $\Phi(n)=\mathbf{P}(N\le n)$, where N is a standard normal random variable. Let $X_1, X_2, \cdots, X_{100}$ be independent identically distributed random variables uniformly distributed on $[1-\sqrt{3}, 1+\sqrt{3}]$. The sample mean is defined as $Y \equiv (X_1+\cdots+X_{100})/100$. Use the central limit theorem to estimate $\mathbf{P}(Y\ge 1.05)$.

n	0.49	0.50	0.51	0.52
$\Phi(n)$	0.6879	0.6915	0.6950	0.6985

8. (15%) Assume that the power consumption rate of a cell phone depends on its battery's voltage level. To be precise, if its battery's voltage level is x volts, then the power consumption rate is $\min(3x, 9)$ mW. It is known that the battery's voltage level is uniformly distributed on [2, 5] volts. The battery stores 1000 joules of electrical energy. (Note: 1 mW = 0.001 joule/sec.)

(a) (7%) What is the cell phone's expected operating time?

(b) (8%) A user has two batteries whose voltage levels are modeled as two independent random variables. He/she would measure the batteries' voltage levels and choose the battery with lower voltage level for the cell phone. What is the cell phone's expected operating time now?