EE 2030 Linear Algebra — Chi-chao Chao

Spring 2011

Homework Assignment No. 2
Due 10:10am, March 30, 2011

`Reading:` Strang, Sections 3.1–3.5.

`Problems for Solution:`

1. Is each of the following subsets of $\mathcal{R}^3$ actually a subspace? If yes, prove it. Otherwise, find a counterexample.

   (a) All vectors (b_1, b_2, b_3) with $b_1 b_2 b_3 = 0$.

   (b) All vectors (b_1, b_2, b_3) satisfy $b_1 + b_2 + b_3 = 0$.

   (c) All vectors (b_1, b_2, b_3) with $b_1 \le b_2 \le b_3$.

2. Suppose S and T are two subspaces of a vector space V. The sum $S + T$ contains all sums $\boldsymbol{s} + \boldsymbol{t}$ of a vector $\boldsymbol{s}$ in S and a vector $\boldsymbol{t}$ in T, i.e.,

$$S + T = \{\boldsymbol{s} + \boldsymbol{t} : \boldsymbol{s} \in S,\ \boldsymbol{t} \in T\}.$$

   The union $S \cup T$ contains all vectors from S or T or both, i.e.,

$$S \cup T = \{\boldsymbol{v} : \boldsymbol{v} \in S \text{ or } \boldsymbol{v} \in T\}.$$

   Determine if each of the following statements is true. If it is, prove it. Otherwise, find a counterexample.

   (a) $S + T$ is a subspace of V.

   (b) $S \cup T$ is a subspace of V.

3. Suppose

$$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{B} \end{bmatrix}.$$

   Find nullspaces $\mathcal{N}(\boldsymbol{A})$, $\mathcal{N}(\boldsymbol{B})$, $\mathcal{N}(\boldsymbol{C})$.

4. Find the reduced row echelon form and the rank of $\boldsymbol{A}$ and $\boldsymbol{B}$:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{bmatrix} 1-d & 2 \\ 0 & 2-d \end{bmatrix}.$$

   (*Hint*: The answers depend on c and d.)

5. Find the complete solution to

$$\begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}.$$

6. Find matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ with the given property or explain why you cannot:

   (a) The complete solution to $\boldsymbol{Ax} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is $\boldsymbol{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

   (b) The only solution to $\boldsymbol{Bx} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is $\boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

7. If $\boldsymbol{w}_1$, $\boldsymbol{w}_2$, $\boldsymbol{w}_3$ are independent vectors, are the vectors $2\boldsymbol{w}_1 + \boldsymbol{w}_2 + \boldsymbol{w}_3$, $\boldsymbol{w}_1 + 2\boldsymbol{w}_2 + \boldsymbol{w}_3$, $\boldsymbol{w}_1 + \boldsymbol{w}_2 + 2\boldsymbol{w}_3$ also independent? You need to justify your answer.

8. (a) Find a basis for the vector space M of all 2 by 3 real matrices whose columns add to zero.

   (b) Find a basis for the subspace of M whose rows also add to zero.