EE 2030 Linear Algebra — Chi-chao Chao
Spring 2011

Homework Assignment No. 4
Due 10:10am, May 4, 2011

`Reading:` Strang, Sections 4.4, Section 8.5, Chapter 5.

`Problems for Solution:`

1. Apply the Gram-Schmidt process to obtain orthonormal vectors $\boldsymbol{q}_1$, $\boldsymbol{q}_2$, $\boldsymbol{q}_3$ from the columns of
$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$
Then write the result in the form $\boldsymbol{A} = \boldsymbol{QR}$.

2. (a) Find orthonormal vectors $\boldsymbol{q}_1$, $\boldsymbol{q}_2$, $\boldsymbol{q}_3$ such that $\boldsymbol{q}_1$, $\boldsymbol{q}_2$ span the column space of
$$\boldsymbol{A} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}.$$
(b) Which of the four fundamental subspaces contains $\boldsymbol{q}_3$?
(c) Find the least squares solution to
$$\boldsymbol{Ax} = \begin{bmatrix} 12 \\ 6 \\ 18 \end{bmatrix}.$$

3. Show that 1, x, and $x^2 - (1/3)$ are orthogonal, when the integration is from $x = -1$ to $x = 1$. Write $f(x) = 2x^2$ as a combination of those orthogonal functions.

4. Find the determinants of the following 4 by 4 matrices by Gaussian elimination:
$$\begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 41 & 42 & 43 & 44 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ t^3 & t^2 & t & 1 \end{bmatrix}.$$

5. This problem shows that the determinant of the following matrix is zero (where the x's are any numbers and they need not be the same):
$$\boldsymbol{A} = \begin{bmatrix} x & x & x & x & x \\ x & x & x & x & x \\ 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & x & x \\ 0 & 0 & 0 & x & x \end{bmatrix}.$$
Explain why all 120 terms are zero in the big formula for det $\boldsymbol{A}$.

6. Let D_n be the determinant of the 1, 1, -1 tridiagonal matrix of order n:

$$D_2 = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}, \quad D_3 = \begin{vmatrix} 1 & -1 & 0 \\ 1 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix}, \quad D_4 = \begin{vmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{vmatrix}, \quad \ldots$$

Expand in cofactors to show that $D_n = D_{n-1} + D_{n-2}$. This yields the *Fibonacci sequence* $1, 2, 3, 5, 8, 13, \ldots$ for the determinants.

7. Use the cofactor matrix to find the inverses of

$$\boldsymbol{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$$

8. Use Cramer's rule to solve each of the following systems of linear equations:

$$\begin{aligned} 2x_1 + x_2 - 3x_3 &= 0 \\ 4x_1 + 5x_2 + x_3 &= 8 \\ -2x_1 - x_2 + 4x_3 &= 2 \end{aligned}$$

and

$$\begin{aligned} x_1 + x_2 &= 0 \\ x_2 + x_3 - 2x_4 &= 1 \\ x_1 + 2x_3 + x_4 &= 0 \\ x_1 + x_2 + x_4 &= 0. \end{aligned}$$