EE 2030 Linear Algebra Chi-chao Chao
Spring 2013

Homework Assignment No. 4
Due 10:10am, May 1, 2013

`Reading:` Strang, Chapter 5.

`Problems for Solution:`

1. (a) By using row operations, compute the determinants of the following matrices:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$

   (b) Find the determinants of $2\boldsymbol{A}$ and $\boldsymbol{A}^T\boldsymbol{B}$.

2. True or false. (If it is true, prove it. Otherwise, find a counterexample.) (All matrices are square matrices.)

   (a) An orthogonal matrix $\boldsymbol{Q}$ has determinant $\det \boldsymbol{Q}$ equal to 1 or -1.

   (b) If $\boldsymbol{A}$ is not invertible, then $\boldsymbol{AB}$ is not invertible.

   (c) The determinant of $\boldsymbol{A} - \boldsymbol{B}$ equals $\det \boldsymbol{A} - \det \boldsymbol{B}$.

   (d) If $\boldsymbol{A}$ is *skew-symmetric*, i.e., $\boldsymbol{A}^T = -\boldsymbol{A}$, then $\det \boldsymbol{A} = 0$.

3. Do Problem 16 of Problem Set 5.2 in p. 265 of Strang.

4. Let S_n be the determinant of the 1, 3, 1 *tridiagonal* matrix of order n:

$$S_1 = \begin{vmatrix} 3 \end{vmatrix}, \quad S_2 = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}, \quad S_3 = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix}, \quad S_4 = \begin{vmatrix} 3 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{vmatrix}, \quad \ldots$$

   (a) Show that $S_n = aS_{n-1} + bS_{n-2}$, for $n \geq 3$. Find the constants a and b.

   (b) Find S_1, S_2, S_3, S_4, and S_5.

5. Do Problem 34 of Problem Set 5.2 in p. 268 of Strang.

6. Use Cramer's rule to solve each of the following systems of linear equations:

$$\begin{aligned} 2x_1 + 5x_2 &= 1 \\ x_1 + 4x_2 &= 2 \end{aligned}$$

and

$$\begin{aligned} 2x_1 + x_2 &= 1 \\ x_1 + 2x_2 + x_3 &= 0 \\ x_2 + 2x_3 &= 0. \end{aligned}$$

7. (a) Find the cofactor matrix $\boldsymbol{C}$ of

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 4 \end{bmatrix}.$$

   (b) Multiply $\boldsymbol{A}\boldsymbol{C}^T$ to find $\det \boldsymbol{A}$.

8. A *Hadamard matrix*, named after the French mathematician Jacques Hadamard, is a square matrix whose entries are either 1 or -1 and *whose rows are mutually orthogonal.*

   (a) Here is an example of a 4 by 4 Hadamard matrix:

$$\boldsymbol{H}_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

   Based on the volume argument, find $|\det \boldsymbol{H}_4|$ (the absolute value of $\det \boldsymbol{H}_4$).

   (b) An 8 by 8 Hadamard matrix can be given by

$$\boldsymbol{H}_8 = \begin{bmatrix} \boldsymbol{H}_4 & \boldsymbol{H}_4 \\ \boldsymbol{H}_4 & -\boldsymbol{H}_4 \end{bmatrix}.$$

   First verify that the rows are mutually orthogonal. Then find $|\det \boldsymbol{H}_8|$.