EE 2030 Linear Algebra — Chi-chao Chao

Spring 2013

**Homework Assignment No. 6**
**Due 10:10am, June 14, 2013**

`Reading:` Strang, Chapter 7.

`Problems for Solution:`

1. Is each of the following transformations linear? If yes, prove it; otherwise, find a counterexample.
   (a) $T(\boldsymbol{v}) = \boldsymbol{v}/\|\boldsymbol{v}\|$.
   (b) $T(v_1, v_2, v_3) = (v_1, 2v_2, 3v_3)$.
   (c) $T(\boldsymbol{v}) =$ largest component of $\boldsymbol{v}$.

2. Consider the vector space $M$ of all 2 by 2 real matrices. The transformation $T : M \to M$ is defined for every $\boldsymbol{X} = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} \in M$ by
$$T(\boldsymbol{X}) = \boldsymbol{AX}$$
where
$$\boldsymbol{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$
   (a) Show that $T$ is linear.
   (b) In class we learned that $\beta = \{\boldsymbol{V}_1, \boldsymbol{V}_2, \boldsymbol{V}_3, \boldsymbol{V}_4\}$ form a basis for $M$, where
$$\boldsymbol{V}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \ \boldsymbol{V}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ \boldsymbol{V}_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \ \boldsymbol{V}_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$
   Find the matrix representation for $T$ (which is a 4 by 4 matrix) in this basis $\beta$.

3. Consider a linear transformation $T : V \to W$. Let $\beta = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ and $\gamma = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \boldsymbol{w}_3\}$ be bases of $V$ and $W$, respectively. Suppose $T(\boldsymbol{v}_1) = \boldsymbol{w}_2$ and $T(\boldsymbol{v}_2) = T(\boldsymbol{v}_3) = \boldsymbol{w}_1 + \boldsymbol{w}_3$.
   (a) Find the matrix representation $[T]_\beta^\gamma$.
   (b) Find the kernel of $T$.
   (c) Find the dimension of the range of $T$.

4. Do Problem 38 of Problem Set 7.2 in p. 398 of Strang.

5. Define the linear operator $T$ on $\mathcal{R}^3$ by

$$T\left(\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right) = \begin{bmatrix} 2v_2 - v_3 \\ 2v_1 + 3v_2 - 2v_3 \\ -v_1 - 2v_2 \end{bmatrix}.$$

Find a basis of $\mathcal{R}^3$ such that the matrix representation for $T$ in this basis is a diagonal matrix.

6. Prove each of the following statements, where $\boldsymbol{A}$ is an $m$ by $n$ matrix and $\boldsymbol{A}^+$ is its pseudoinverse.

   (a) $\boldsymbol{A}\boldsymbol{A}^+\boldsymbol{A} = \boldsymbol{A}$.

   (b) $(\boldsymbol{A}^+\boldsymbol{A})^2 = \boldsymbol{A}^+\boldsymbol{A}$.

7. (This problem counts double.) Consider the matrix

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}.$$

   (a) Find the singular value decomposition of $\boldsymbol{A}$.

   (b) Find the pseudoinverse $\boldsymbol{A}^+$ of $\boldsymbol{A}$.

   (c) Find the projection matrix onto the row space of $\boldsymbol{A}$.

   (d) Find a right inverse of $\boldsymbol{A}$.

   (e) Find the shortest leat squares solution to $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, where

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}.$$