EECS 2060 Discrete Mathematics — Chi-chao Chao
Spring 2021

Homework Assignment No. 4
Due 10:10am, May 12, 2021

Reading: Grimaldi: Sections 9.3 Partitions of Integers, 5.7 Computational Complexity, 5.8 Analysis of Algorithms, 11.1 Definitions and Examples, 11.2 Subgraphs, Complements, and Graph Isomorphism, 11.3 Vertex Degree: Euler Trails and Circuits (up to Example 11.12).

Problems for Solution:

1. (a) Let a_n denote the number of ways in which the sum n will show when four dice are rolled, for $n \geq 0$. Find the generating function for a_n.

   (b) Let b_n denote the number of ways in which the sum n will show when four dice are rolled with the first, third ones showing odd and the second, fourth ones showing even, for $n \geq 0$. Find the generating function for b_n.

2. Find the generating functions for the sequences of the numbers of partitions of the nonnegative integer n with the following properties:

   (a) $p(n \,|\, \text{each part appears an even number of times})$.

   (b) $p(n \,|\, \text{each part is even})$.

3. Show that the number of partitions of the positive integer n where no part is divisible by 4 is equal to the number of partitions of n where no even part is repeated (although odd parts may or may not be repeated).

4. Show that the number of partitions of the positive integer n is equal to the number of partitions of $2n$ into n parts.

5. The following pseudocode procedure can be used to evaluate the polynomial

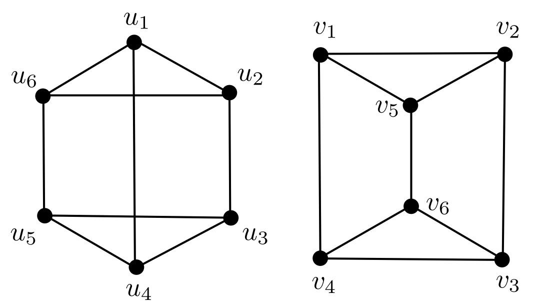
$$a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n$$

   at $x = r$ by using *Horner's method.*

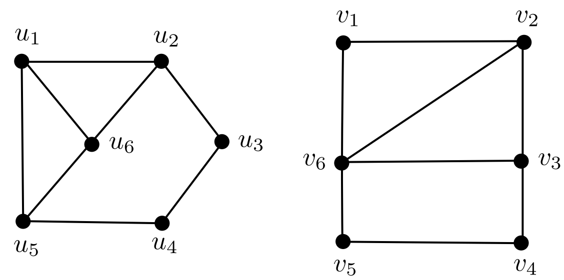
   procedure polynomial evaluation
   begin
   $value := a_n$
   for $i = 1$ **to** n **do**
       $value := a_{n-i} + r * value$
   end

   Compute the numbers of additions and multiplications which take place in the evaluation of the polynomial. Then estimate them using the big-O notations.

6. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide an argument that none exists.



(a)

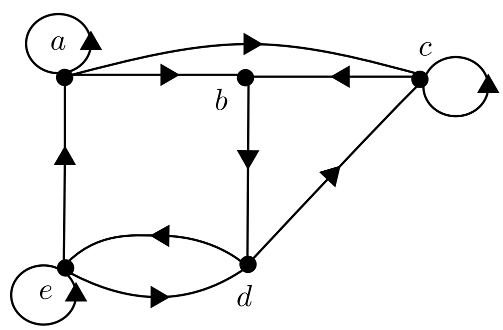


(b)

7. In a directed graph (simple graph or multigraph) $G = (V, E)$, the *in degree* of a vertex $v \in V$, denoted by $\deg^-(v)$, is the number of edges with v as their terminal vertex. The *out degree* of v, denoted by $\deg^+(v)$, is the number of edges with v as their initial vertex. (Note that a loop around a vertex contributes 1 to both the in degree and the out degree of this vertex.)

   (a) Show that
$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|.$$

   (b) Consider the following directed graph. First find the corresponding adjacency matrix. Then verify the result in (a).



8. The *complement graph* $\overline{G}$ of an undirected simple graph G has the same vertices as G. Two (distinct) vertices are adjacent (i.e., linked by an edge) in $\overline{G}$ if and only if they are not adjacent in G.

   (a) If G has n vertices and their degrees are d_1, d_2, ..., d_n, what are the degrees of the vertices of $\overline{G}$?

   (b) If a graph G has 9 edges and its complement graph $\overline{G}$ has 6 edges, then how many vertices does G have?

Homework Collaboration Policy: I allow and encourage discussion or collaboration on the homework. However, you are expected to write up your own solution and understand what you turn in. Late homework is subject to a penalty of 5% to 40% of your total points.